APPENDIX VII

Wedge Analysis

1. General. The procedures presented in this appendix assume that shear failure may occur in an embankment and its foundation along a surface approximated by a series of planes. These procedures are variations of what is generally termed the wedge method of analysis. This method is particularly applicable to a zoned embankment containing cohesionless outer shells and a relatively thin core resting on either homogeneous or stratified foundation materials. The analyses presented in this appendix emphasize the application of the wedge method to embankments having impervious cores with gravel or rock shells and demonstrate the influence of the location of the core on embankment stability. Examples are given for embankments with central impervious cores, and for embankments with inclined impervious cores located within the upstream portions of the embankments. The planes defining the boundaries of the sliding mass that are shown in the examples are not necessarily the most critical failure planes, since the examples are presented only to illustrate the procedures involved.

2. Basic Principles. In the wedge method, the soil mass is usually divided into three segments: an active wedge, a central block, and a passive wedge, as shown in figure 1 of plate VII-1. Vertical boundaries are assumed between the central block and the active and passive wedges. The forces on each segment are considered separately as shown in figures 2 through 4 of plate VII-1. The developed values of cohesion and angle of internal friction along the failure surfaces are controlled by the assumed trial factor of safety, F.S., so that

$$c_D = c/F.S.$$

$$\tan \phi_D = (\tan \phi)/F.S.$$

Consequently, the magnitudes of the resultant earth forces E_A and E_P also depend on the trial safety factor. The resultant earth forces acting at the vertical boundaries of the passive and active wedges are determined by constructing force polygons, as illustrated in figures 2 and 3, respectively, of plate VII-1, and are then incorporated in the force polygon for the central block (fig. 4). A condition of equilibrium will generally not be obtained on the first trial and several trial analyses with different safety factors are required. In each analysis, the force necessary to close the polygon (fig. 4, plate VII-1) is denoted as ΔE_H. The force ΔE_H is assumed to act horizontally and its magnitude and sign vary with the trial factor of safety. A plot is made of ΔE_H versus the trial factors of safety, as shown in figure 5, plate VII-1, to determine the factor of safety at which ΔE_H is zero. This factor of safety is that required to balance the forces for the sliding surface being analyzed. Various trial locations of the active and passive wedges are required to determine the minimum safety factor.

3. Basic Criteria. Criteria for selecting the direction of the active and passive earth forces are illustrated in plate VII-2. However, these criteria are illustrative only and should be modified where differential foundation settlement resulting from consolidation of soft layers or from a variable subsoil profile makes this desirable. The criteria shown in plate VII-2 apply only where the maximum settlement will occur beneath the center of the embankment. The location of the critical sliding planes is often controlled by weak zones, such as a foundation layer and/or an inclined impervious core, and must be determined by trial. In general, sliding will occur near the bottom of a weak layer. In the discussions that follow, a thin weak layer has been assumed.

a. Active Earth Forces. (1) A general rule for selecting the direction of E_A is shown in the tabulation in figure 1 of plate VII-2. When the sliding surface lies in cohesive materials or includes a portion of the crest or reverse slope (plate VII-2), the maximum value of E_A must be determined by trial force polygons using various values of θ_A. As a first trial, θ_A can be assumed equal to $45^o + (\phi/2)$. When the sliding plane is located within a thin inclined core, the slope of the core will generally govern the angle of the sliding plane.

(2) The maximum value of E_A and corresponding value of θ_A can be determined using the conjugate stress procedure illustrated in plate VII-3 when (a) the active sliding surface is in cohesionless materials, (b) the entire active wedge is under the slope, (c) E_A acts parallel to the outer slope (fig. 1(b), plate VII-2), and (d) seepage forces are not present. The active earth force may also be computed by obtaining the earth pressure coefficient K_A from earth pressure tables[14] using the value of ϕ_D and the assumed angle of the active earth force as the angle of wall friction.

(3) When the active wedge is composed of different materials (fig. 1(c), plate VII-2), the angles of the active sliding surfaces depend on the shear strengths of the soils involved. However, in preliminary design analyses for dams and for design of levees, channels, miscellaneous embankments, and other structures, the active sliding surface can be assumed to be inclined at $45^o + \phi/2$ for each material. For final design analyses of dams and for design of more critical earth structures, θ_A should be varied within each soil zone through which the active sliding surface passes until the maximum value of E_A is found. To determine the magnitude of the resultant active force, the wedge must be subdivided as shown in figure 1(c) and the total earth force at each boundary determined as shown by the force polygon. The direction of the resultant forces E_{A1}, E_{A2}, and E_A are assumed to be in accordance with the general rule given in plate VII-2. Other trial locations of the plane ac are necessary in all analyses to determine the lowest factor of safety.

b. Central Block. (1) Where the failure plane beneath the central block passes through more than one material or where the failure plane passes through a single material but a different shear strength is used because of changing normal stress (e.g. using a composite S and R envelope), the central block should be broken up into its component parts based on material type (or shear strength) as described previously for the active wedge.

Resultant forces acting on boundaries between these "subblocks" can be assumed to be inclined at any value intermediate between the inclination of E_A and E_P, but are more conveniently assumed to be horizontal. With this latter assumption, the normal stress on the failure plane is equal to the overburden stress.

(2) The case should also be considered where a horizontal failure surface parallels a boundary between different materials (for example, a clay stratum overlying or underlying cohesionless material). In such a case, the lowest shear resistance along this failure surface may be when sliding is partially in one material and partially in the other; this occurs because sliding in the cohesionless layer may offer less shear resistance than in the clay under low effective normal stresses, whereas under high effective normal stresses the reverse may be true. The point at which this "switch" occurs can easily be determined by computing the normal stress at which the strength envelopes for the two materials intersect.

c. Passive Earth Forces. (1) When the passive wedge is near the toe of the embankment, as in the case shown in figure 2(a), plate VII-2, in which sliding is assumed to occur along a weak plane within the foundation, the direction of E_P is assumed to be horizontal. The passive wedge will usually be separated from the active wedge by a central block, and trial locations of the vertical boundary between the passive wedge and central block are required to determine the lowest factor of safety, as illustrated in figure 2(a). Where the passive wedge is located in cohesionless material and the vertical boundary is at the toe of the embankment (wedge A in fig. 2(a), plate VII-2), the resultant passive soil resistance E_P can be determined graphically or from the equation

$$E_P = 1/2\, \gamma\, h^2 K_P$$

in which

$$K_P = \frac{1 + \sin \phi_D}{1 - \sin \phi_D}$$

When the vertical boundary is not at the toe of the embankment, trial values of θ_P must be assumed for each trial factor of safety until a minimum value of E_P is obtained. When the passive wedge includes several soil zones, θ_P should be varied and the criteria in paragraph 3a(3) applies.

(2) Where sliding is assumed to occur along the ground surface as shown in figure 2(b), plate VII-2, the inclination of E_P is assumed to be the same as that of E_A. If a central block is present, E_P acts parallel to the outer slope. The magnitude of E_P is determined from force polygons for various trial factors of safety. When the embankment material is cohesionless and the foundation is stronger than the embankment, a passive sliding plane is assumed to intersect the toe of the embankment and make an angle of θ_P with the horizontal (fig. 2(c), plate VII-2). In this case, E_P acts parallel to the outer slope and the conjugate stress procedure (plate VII-3) may be used to determine θ_P and E_P for each trial factor of safety.

(3) Examples of the criteria above and procedures for handling water forces for various design cases are described in the following paragraphs.

4. <u>End of Construction--Case I.</u>† The end-of-construction stability of an embankment composed of a granular shell and impervious cohesive core is influenced by the core location. Accordingly, examples for embankments with both central and inclined cores are presented. Unit weights and shear strengths should correspond to those expected at the end of construction, as discussed in paragraphs 9 and 11a of the main text. In the analysis, S shear strengths are used for free-draining embankment and foundation materials and Q strengths are used for impervious core or foundation soils. The R strengths may be used for relatively thin clay strata in the foundation when consolidation will be essentially complete at the end of construction. In some cases, it may be necessary to use a design strength intermediate between Q and R. Additional analyses should be made during construction of the embankment, as discussed in Appendix VIII.

† Case designations are described in paragraph 11 of the main text.

a. Embankment with Central Core. (1) Where the foundation strength is equal to or greater than the strength of a cohesionless embankment shell flanking a narrow central core, the safety factor can be estimated using the infinite slope equation $F.S. = \frac{\tan \phi}{\tan \beta}$, as discussed in Appendix V.

(2) For conditions where the foundation contains a layer that is weaker than the shell, the factor of safety must be found by trial. This condition is illustrated in plate VII-4. The assumed failure mass is divided into an active wedge, a central block, and a passive wedge. A trial point 1 is selected for the upper end of a series of active wedges corresponding to various trial factors of safety. The earth force E_A and the inclination θ_A of the active sliding plane can be determined for each trial safety factor according to the conjugate stress procedure, since the earth force E_A is assumed to be parallel to the outer slope. A simplified conjugate stress procedure for determining K_A and θ_A is shown in figure 2 of plate VII-4. The direction of the earth force E_P is assumed to be horizontal. Because the upper surface of the passive wedge in the example is horizontal, the passive pressure coefficient K_P is that given in figure 2. The computation of the passive force E_P is also given in this figure. When several types of material are contained within the active or passive wedges, E_P and E_A can be determined from force diagrams.

(3) Using the values above for E_A and E_P, a force polygon for the central block can be constructed as shown in figure 3 of plate VII-4. The polygon does not close by the force ΔE_H. A plot of ΔE_H versus trial factors of safety is used as shown in figure 4, plate VII-4, to obtain the factor of safety when ΔE_H is zero and the force polygon closes. Other trial locations of the active and passive wedges should be used to find the minimum safety factor. When a portion of the active plane passes through the core, E_A is determined by trial by constructing a force polygon as shown in figure 1(c), plate VII-2.

b. Embankment with Inclined Core. (1) The failure surface for this condition will normally be located in the lower strength core material. While the zone of minimum strength is probably near the middle of the core, because consolidation takes place at a slower rate here than at the outer faces, the failure surface is normally assumed to lie along the downstream boundary where the largest driving force is obtained. If the foundation is as strong as or stronger than the shell, the lower portion of the trial failure surface will be entirely in the shell. This case is illustrated in plate VII-5.

(2) In the embankment section shown in figure 1, plate VII-5, the toe of the passive wedge is assumed to coincide with the outer toe of the dam. The inclination of the base of the passive wedge and the magnitude of the earth force E_P are determined from the conjugate stress assumption, as discussed in paragraph 3c of this appendix and as shown in figure 2, plate VII-5, for a trial safety factor of 1.5. When the trial sliding plane of the active wedge is along the boundary of two embankment zones, the trial sliding surface plane should be located in the material having the lower developed shear strength so that the maximum resultant active earth force is obtained. In the case shown in plate VII-5, the S shear strength of the material in downstream gravel filter is less than the Q shear strength of the core under low normal stresses, but the reverse is true under higher loads; therefore minimum resistance is obtained when the upper portion of the sliding surface is in the filter and the lower portion is in the core. A method of locating the break point is illustrated in figure 1 of plate VII-5. Several trial locations (A, B, and C in fig. 1) are selected, and the weight of the active wedge to the right of each location is determined. A force polygon is constructed at each trial location using the developed shear strength of each material; the developed Q strength of the core and the developed S strength of the gravel filter are used in the case of the example in plate VII-5. The intersection of the friction vector for the developed S strength $F_{A(S)}$ with the E_A vector is located for each polygon, and a smooth curve is drawn through these points. A similar curve is drawn through the intersections of E_A and $F_{A(Q)}$ vectors. The intersection of the two curves locates the point where the two shear strengths result in the same value of E_A (point D in fig. 1).

From this point, a line parallel to the S strength friction vector $F_{A(S)}$ is drawn to the sliding surface (dashed line from point D to point E in fig. 1 of plate VII-5). This locates point E on the sliding surface, to the right of which the plane of sliding would lie in the gravel filter and to the left of which sliding would occur in the core. The force polygons for the active wedge and central block are shown in figures 3 and 4, respectively. The forces ΔE_H required to close the force polygons for the central block are plotted versus trial safety factors in figure 5, where the factor of safety to balance forces for the sliding surfaces analyzed is shown to be 1.62.

(3) If the foundation has a lower shear strength than the embankment shell, the trial failure surface will pass through the foundation.

5. Sudden Drawdown--Cases II and III. Appropriate unit weights, shear strengths, and design assumptions to be used in sudden drawdown analyses are described in paragraph 11b of the main text. In the wedge method, the active and passive forces are influenced by seepage forces when materials in the shell are semipervious.

a. Embankment with Central Core. (1) Sudden drawdown is not generally critical for embankments having free-draining shell materials and a narrow central core, and this case need not be analyzed unless a relatively weak layer is present in the foundation. The safety factor of free-draining cohesionless shell materials can be approximated using the infinite slope method described in Appendix V. However, detailed stability analyses are required when the upstream shell is composed of sands or gravels of low permeability. If the foundation contains a thin layer that is not as strong as the shell, the horizontal portion of the trial sliding surface will pass through the weaker foundation layer, as illustrated in figure 1, plate VII-6, for an embankment having semipervious shells. The potential failure mass is divided into a passive wedge, a central block, and an active wedge. Because the shell material is semipervious, it may be necessary to construct a drawdown flow net to evaluate the seepage forces. Various trial locations of the boundaries between the wedges and the central block and various inclinations of the active and passive sliding planes must be assumed.

(2) In the example shown in plate VII-6, the boundary between the passive wedge and central block is assumed to be at the toe of the embankment; the computations for E_P are shown directly below figure 1. A trial location with θ_A = 33.5 deg is assumed for the active sliding plane.

(3) The use of the R or S shear strengths along the trial sliding planes is established by comparing the normal stress at the inflection point of the composite shear strength envelopes shown in figure 2 with the approximate effective normal stresses along the trial failure planes. The procedure for doing this is demonstrated in plate VII-6.

(4) The force polygons for the active wedge and for the central block are shown in figure 3 of plate VII-6 for a trial factor of safety of 1.3. Various safety factors are tried until a balance of forces is obtained. A plot of ΔE_H versus trial factor of safety is shown in figure 4 of plate VII-6 for the trial locations of the active and passive wedges. Other trial locations are required to determine the minimum factor of safety. A check of the lower (1 on 3.5) portion of the outer embankment slope, using the equation for horizontal flow given in Appendix V, results in a factor of safety of 1.28; the factor of safety for the upper 1-on-3 slope ranges from 1.07 for horizontal flow to 1.17 for flow parallel to the outer slope, with an average factor of safety of 1.12. Therefore, the surface of the outer slope has a low factor of safety for sudden drawdown as compared to a failure surface through the embankment and the weak foundation. If there is an appreciable thickness of riprap on the outer slope, the weight of riprap should be taken into consideration in the analysis.

b. Embankment with Inclined Core. (1) The sliding surface in the inclined core is assumed to be located along the boundary between the core and the upstream shell because the shear strength of this portion of the core is not increased by seepage forces prior to drawdown. However, stability should also be checked with the sliding surface at the downstream boundary of the core, assuming that the core along the sliding surface is fully consolidated under the weight of overlying material and by seepage forces. When the foundation is stronger than the embankment, the failure mass consists of an active and a passive wedge, with the toe of the passive wedge coinciding with the toe of the embankment as shown in figure 1, plate VII-7. The inclination of the base of the passive wedge θ_P and the passive force E_P are determined using the conjugate stress assumption as shown in figure 3. The most critical condition for each trial factor of safety is obtained with the passive wedge completely submerged, and thus the critical lowered pool level for each trial factor of safety should be located to intersect the upstream slope at the top of the vertical boundary between the active and passive wedges. If the location of the estimated actual drawdown pool level is above or below the critical lowered pool level, the factor of safety will be slightly higher than that for the critical lowered pool level.

(2) In evaluating the active force E_A (fig. 4, plate VII-7), the frictional force F_A is based on the submerged weight of the rock fill and filter (W_{A1} and W_{A2}) below the maximum pool level and the moist weight of the rock fill and filter (W_{A3}) above this level. During sudden drawdown, the upstream shell above the low pool level changes in weight from submerged to moist. It is assumed that this added increment of weight induces pore pressure, but does not cause any immediate gain of shear strength in the core. The induced pore pressure force created by the difference between the moist and submerged weights is represented in the force polygon in figure 4 by U_A This force need not be explicitly computed, as can be seen from the force polygon. Figure 4 shows that the resultant of U_A and the change in weight of the shell (492 kips) contribute a major portion of E_A

(3) Curves of E_P and E_A for various trial factors of safety are presented in figure 5, plate VII-7. A condition of equality between E_P and E_A for the sliding surface analyzed exists for a factor of safety equal to 1.23 in the case illustrated.

(4) If the shell is stronger than the foundation, the passive sliding plane will be in the foundation and full drawdown should be considered. If high

tailwater conditions will exist during spillway operations, a check of the downstream toe for sudden drawdown should be made.

6. Partial Pool, Upstream Slope--Case IV. A static reservoir reduces the stability of the upstream slope because of reduction in weight and resistance of the passive wedge due to buoyancy. In many cases, a pool elevation above conservation pool elevation is critical; this critical elevation must be determined by trial. Basic assumptions and shear strengths for this case are described in paragraph 11c of the main text.

a. Embankment with Central Core. The procedure used is similar to that discussed in paragraph 4a of this appendix, except that a horizontal saturation line is assumed within the embankment at the trial level of the pool. Either the S or $\frac{R+S}{2}$ shear strength of the core is used, depending on the magnitude of the effective normal stress.

b. Embankment with Inclined Core. (1) A stability analysis for an embankment with an inclined core on a strong foundation is shown in plate VII-8. The embankment section is shown in figure 1 of the plate. The inclination of the passive sliding plane θ_P and the passive earth force E_P for a trial factor of safety of 1.5 are determined as shown in figure 2. As in the sudden drawdown case, the most critical condition for each trial factor of safety is obtained with the passive wedge completely submerged, and thus the lowered pool level for each trial factor of safety should be located to intersect the upstream slope at the top of the vertical boundary between the active and passive wedges. Submerged weights are used below the partial pool elevation and moist unit weights above.

(2) Computations to the right of figure 1, plate VII-8, illustrate a simplified procedure for computing normal stresses on the trial failure planes for determining use of S or $\frac{R+S}{2}$ strengths. Composite strength envelopes are shown in figure 3.

(3) The value of E_A is determined from a force polygon as shown in figure 4, plate VII-8. The comparison of E_A and E_P versus trial factor of safety, shown in figure 5, indicates that the factor of safety for the sliding

surfaces analyzed is 1.51.

(4) This case should also be analyzed assuming the active sliding plane at the downstream face of the core with the pool level at several locations to check for a more critical condition. The analyses should assume that the core is consolidated under the overlying weights corresponding to the critical pool elevation.

(5) If the foundation is weaker than the shell, the passive sliding plane will be in the foundation, and the passive resistance is determined using a central block in a manner similar to that shown in plate VII-6.

7. Steady Seepage with Maximum Storage Pool--Case V. Steady seepage reduces the weight of the soil mass below the saturation line by hydrostatic uplift, and thus frictional shearing resistance is reduced. At the same time, the water forces of the reservoir pool act horizontally against the impervious core in the downstream direction. Basic criteria and shear strength to use are discussed in paragraph 11d of the main text.

a. Embankment with Central Core. (1) If the core is narrow with steep slopes and the embankment rests on a strong foundation, only the stability of the downstream shell need be examined. If the shell material is cohesionless and free draining, the critical sliding surface is the slope of the outer shell, and the factor of safety can be expressed as

$$F.S. = b \tan \phi$$

where

b = cotangent of the downstream embankment slope

ϕ = angle of internal friction of the shell material

Where cores are wide or foundations are weaker than the shell, the most critical sliding surfaces may pass through these zones and must be found by trial. Where the shear strength of the foundation is less than that of the shell material, the weakest horizontal sliding surface may be either in the shell just above the foundation, slightly within the foundation layer, or at the bottom of the foundation layer, depending upon the normal loads and shear strengths. An example is given in plate VII-9. The active wedge and central block are divided into intermediate sections at boundaries where the shear strength parameters change. Composite strength diagrams are shown in figure 2, and computations to determine where the S or $\frac{R + S}{2}$ strength should be used are given.

(2) Since the active wedge portion A1, located in the cohesionless shell, is not entirely submerged, the maximum value of the active resultant force E_{A1} must be determined graphically (fig. 3, plate VII-9) based on the weight W_{A1} and direction of F_{A1} for each trial factor of safety. Values of θ_{A1} can be determined from plate VII-11 or by trial. However, θ_{A1} varies only slightly for the trial factors of safety used in the example, and a value of 65 deg is used for all trial factors of safety. When that portion of the active wedge in cohesionless material is completely submerged (or completely above the seepage line) E_{A1} can be computed using the chart in plate VII-12. The determination of the hydrostatic forces is shown in figure 1, and the values of E_A (for θ_{A2} = 50 deg) and E_P are shown in figures 4 and 5, respectively, of plate VII-9.

(3) The magnitude of E_A for each trial safety factor varies with the assumed inclination of the base of the active wedge θ_{A2} which must be varied to obtain the maximum value of E_A. A plot of E_A and E_P versus trial factors of safety is shown in figure 6, plate VII-9. It should be noted from figure 6 that θ_{A2} for the lowest factor of safety is 60 deg.

b. Embankment with Inclined Cores. The steady seepage case is not critical for an embankment with an inclined upstream core on a strong foundation. Conditions existing either at the end of construction or sudden drawdown are usually the critical cases for such a design.

8. Steady Seepage with Surcharge Pool--Case VI. This case applies after a condition of steady seepage has been established at a given pool level, the reservoir pool quickly rises to the surcharge pool level, and no appreciable change in the seepage pattern takes place because of the short duration at the

higher level. This analysis is especially applicable to rock-fill dams having narrow central cores. The procedure of analysis and shear strength criteria used for this case are identical to those given for Case V; the only difference in the two analyses is that in Case VI the horizontal thrust from the surcharge pool is added to the active wedge force polygon and the unit weight of that portion of the pervious upstream zone between the surcharge pool and the storage pool becomes submerged instead of moist. An example of this analysis is given in plate VII-10 where a surcharge pool has been applied to the steady seepage example shown in plate VII-9.

9. Earthquake. As discussed in paragraph 11f of the main text, it is assumed that the earthquake imparts an additional horizontal force F_h acting in the direction of sliding of the potential failure mass. This force is equal to the total weight of the sliding soil mass W times the seismic coefficient ψ. The weight W is based on the saturated unit weight below and moist unit weight above the saturation line, but does not include the weight of water above the embankment slope. In the wedge analysis, horizontal seismic forces are computed individually for the active wedge, the passive wedge, and the central block, and included in the respective force polygons.

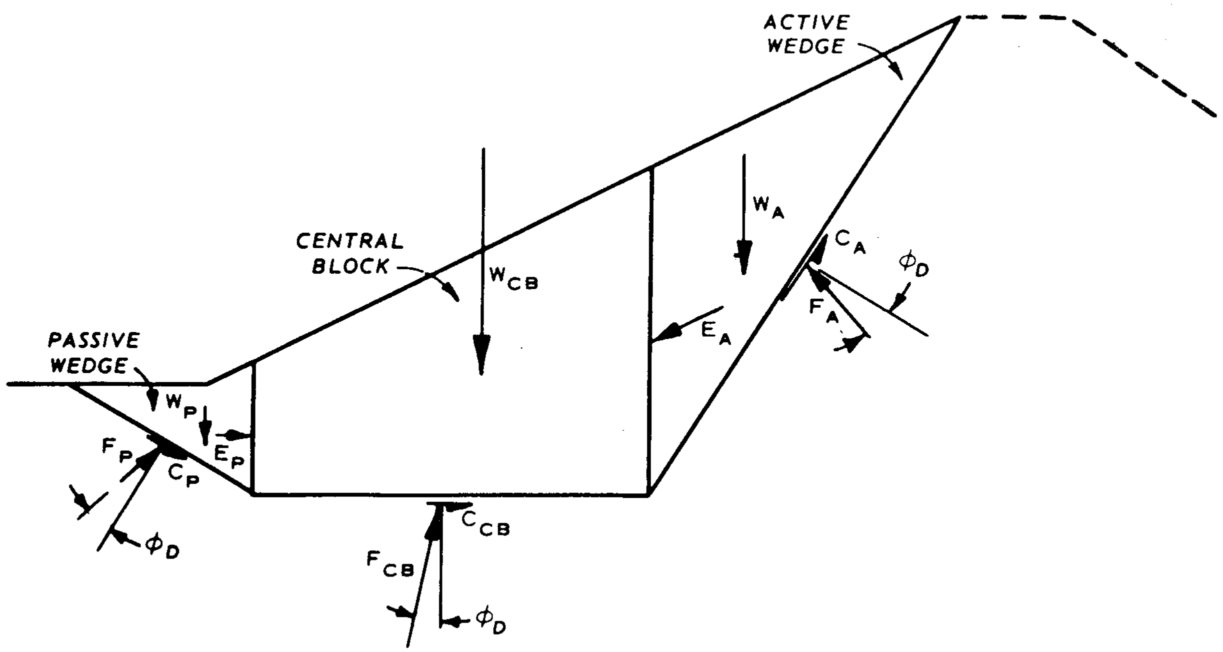


FIGURE 1. EMBANKMENT SECTION

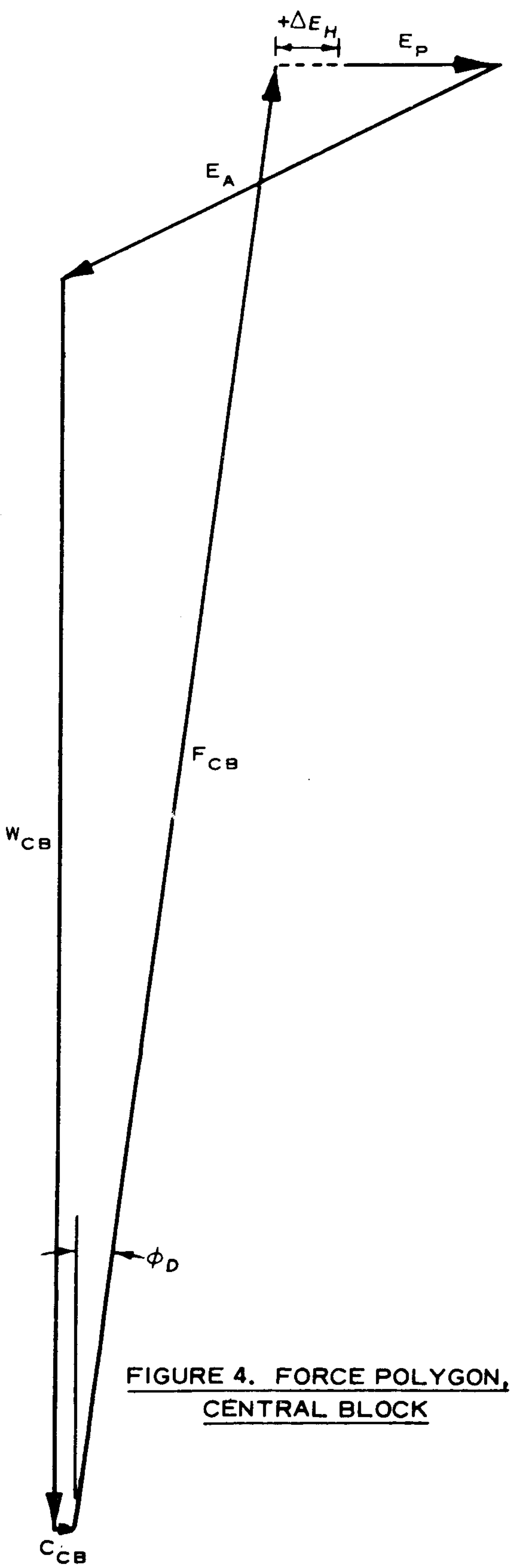


FIGURE 4. FORCE POLYGON, CENTRAL BLOCK

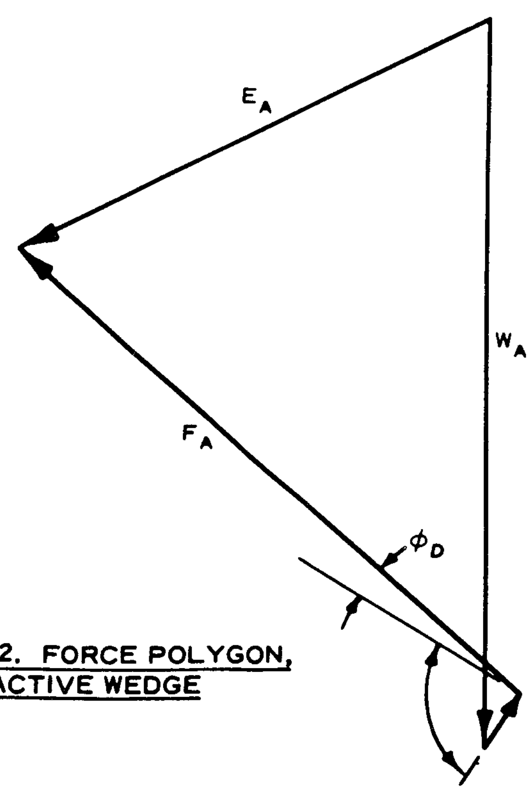


FIGURE 2. FORCE POLYGON, ACTIVE WEDGE

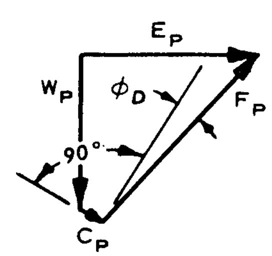


FIGURE 3. FORCE POLYGON, PASSIVE WEDGE

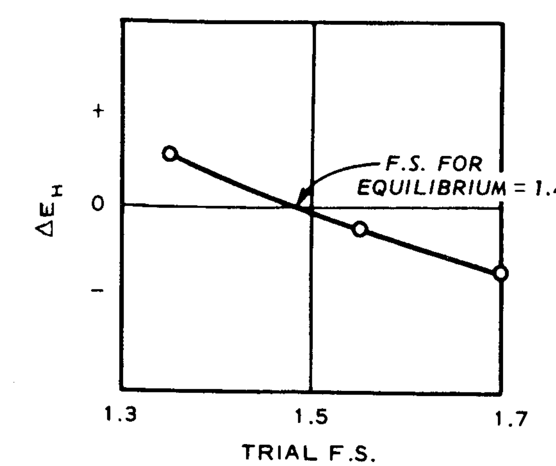
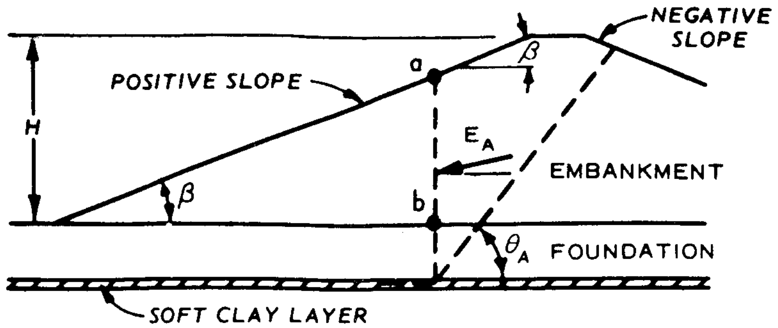
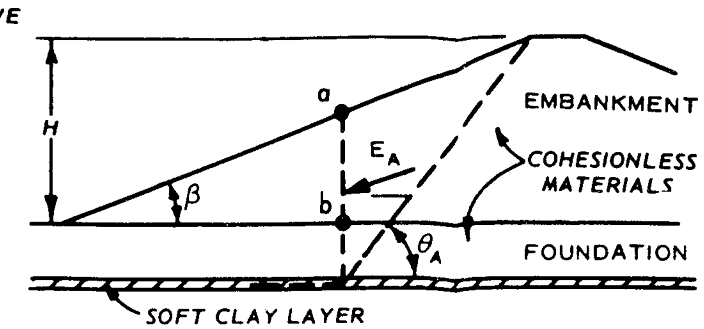


FIGURE 5. TRIAL FACTOR OF SAFETY VS ΔE

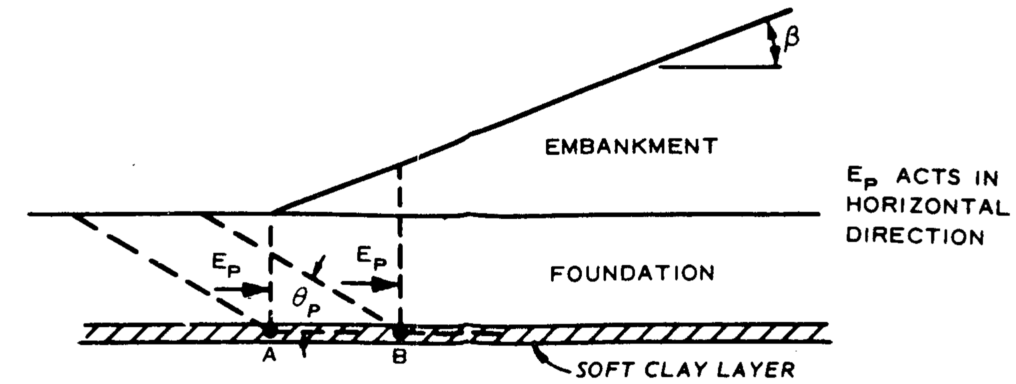
STABILITY ANALYSIS WEDGE METHOD



(a) ACTIVE WEDGE IN COHESIVE MATERIAL OR PARTIALLY UNDER CREST IN COHESIONLESS MATERIAL. (SOLVE FOR E_A USING FORCE POLYGON.)



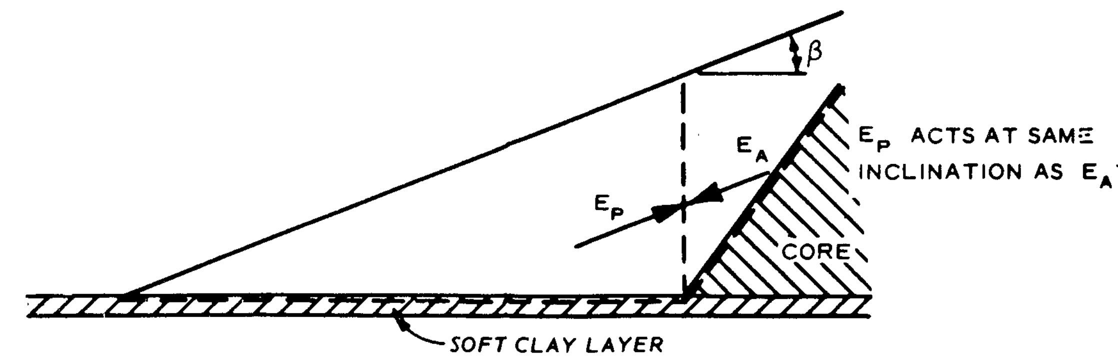
(b) ACTIVE WEDGE IN COHESIONLESS MATERIAL AND ENTIRELY UNDER SLOPE. (SOLVE FOR E_A USING CONJUGATE STRESS ANALYSIS.)



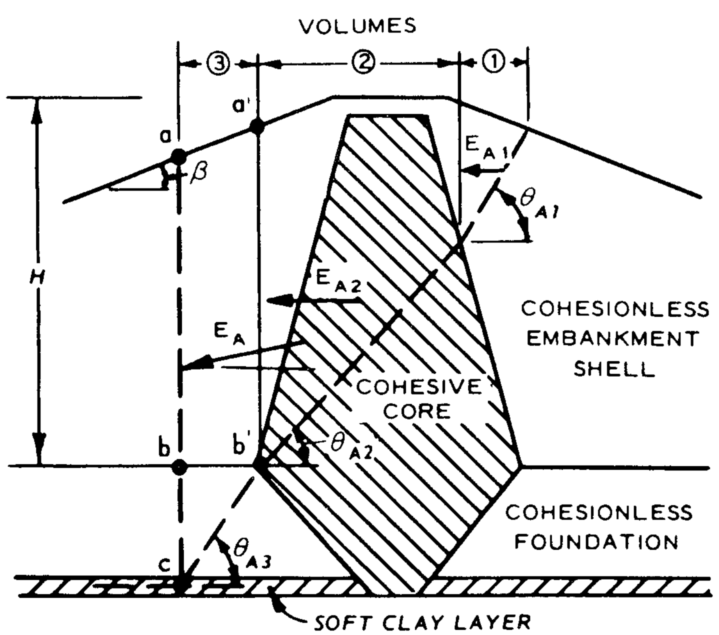
(a) SLIDING IN FOUNDATION

INCLINATION OF E_A FOR ALL CASES (POSITIVE SLOPE*)		
	ANGLE OF E_A WITH HORIZONTAL	
HEIGHT, $\overline{ab}$	SANDS AND SILTS	CLAYS
ab < 5/8 H	β	β/2
5/8 H ≤ ab ≤ 7/8 H	β/2	β/2
ab > 7/8 H	0	0

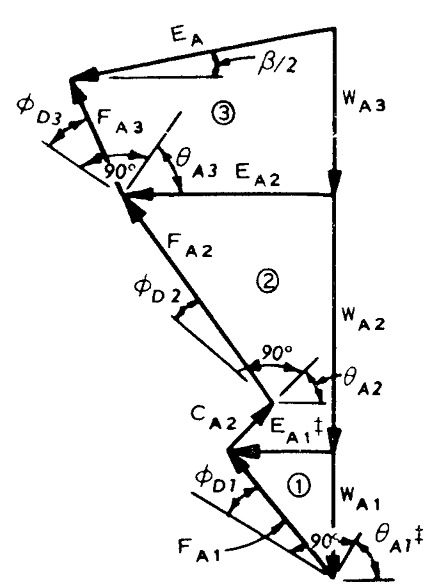
* E_A HORIZONTAL FOR SEGMENT UNDER NEGATIVE SLOPE



(b) SLIDING AT GROUND SURFACE

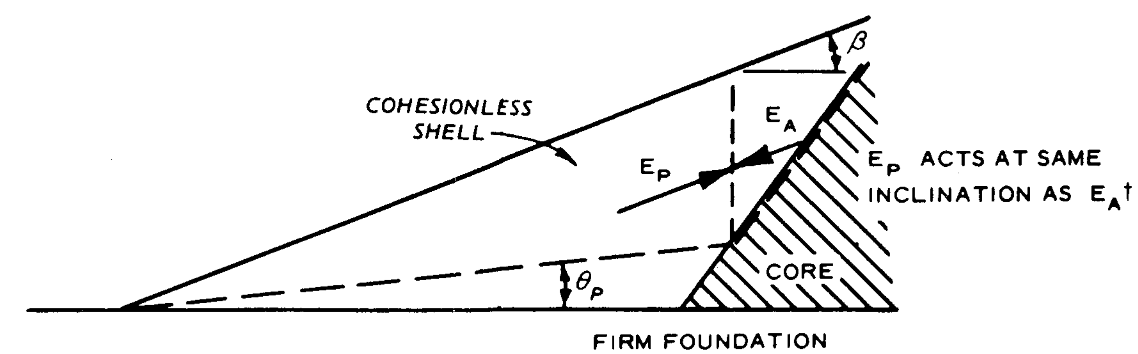


(c) ACTIVE WEDGE INCLUDES DIFFERENT MATERIALS. (SOLVE FOR E_A USING FORCE POLYGON.)



‡ FOR HOMOGENEOUS SANDS AND SILTS, E_{A1} AND θ_{A1} CAN BE DETERMINED DIRECTLY FROM PLATES VII-11 AND VII-12.

FIGURE 1. ACTIVE WEDGES



(c) SLIDING IN EMBANKMENT

FIGURE 2. PASSIVE WEDGES

† HOWEVER IF THERE IS A CENTRAL BLOCK, E_P ACTS PARALLEL TO THE OUTER SLOPE.

DIRECTION OF RESULTANT EARTH FORCES AND ACTIVE AND PASSIVE SLIDING PLANES, WEDGE METHOD

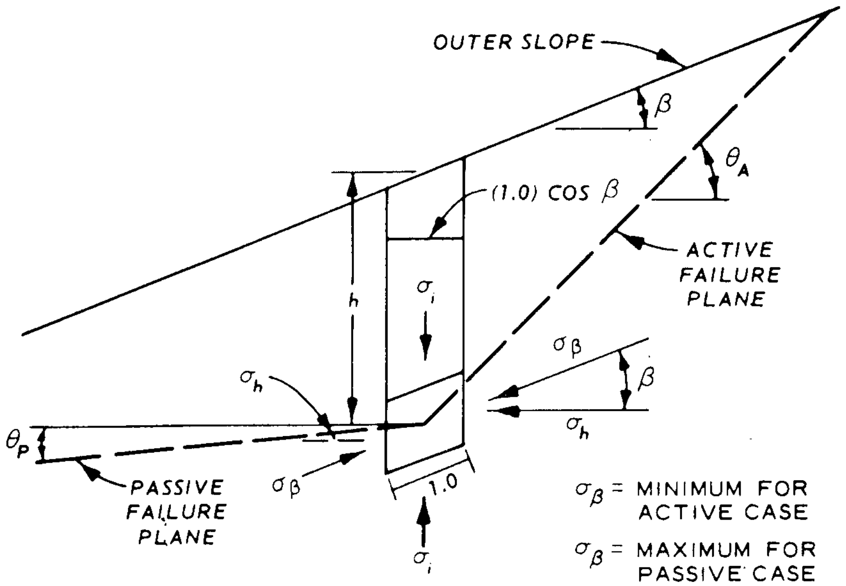


FIGURE 1. CONJUGATE STRESSES, COHESIONLESS EMBANKMENT, NO SEEPAGE FORCES

σ_i = VERTICAL STRESS ON PLANE PARALLEL TO OUTER SLOPE = $\gamma h \cos\beta$ (WHERE γh = VERTICAL STRESS ON HORIZONTAL PLANE AT DEPTH h)

σ_h = HORIZONTAL STRESS ON VERTICAL PLANE = $\sigma_\beta \cos\beta$

K = EARTH PRESSURE COEFFICIENT = $\dfrac{\sigma_h}{\sigma_i} = \dfrac{\sigma_\beta \cos\beta}{\gamma h \cos\beta}$

SINCE σ_β AND σ_i ARE EACH PARALLEL TO THE PLANE OF THE OTHER, THEY ARE CONJUGATE STRESSES.
AN INFINITE SLOPE IS ASSUMED, AND σ_i IS CONSTANT AT DEPTH h ALONG THE SLOPE.

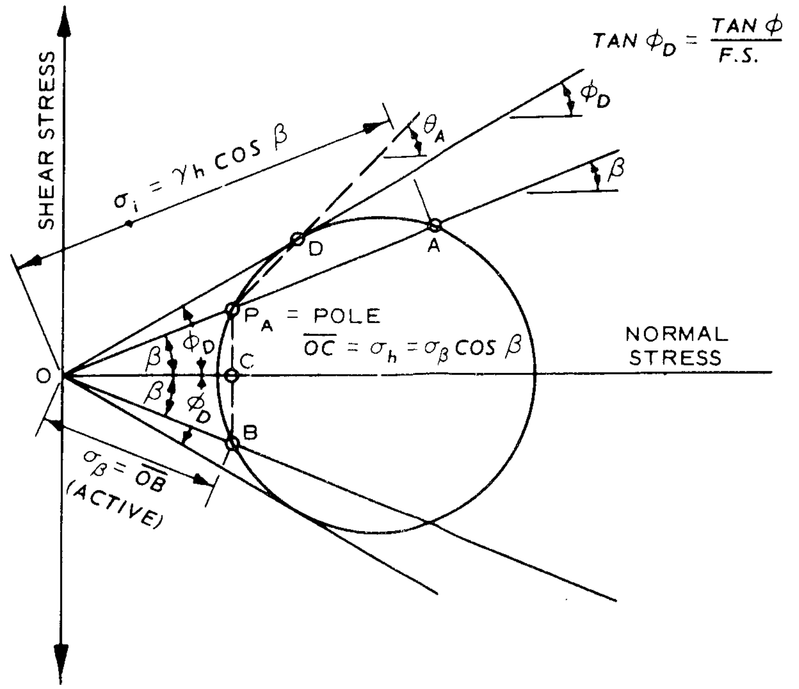


FIGURE 2. PROCEDURE FOR DETERMINING K_A & θ_A, ACTIVE CASE (σ_β = MINIMUM)

1. CONSTRUCT LINES AT ANGLES ϕ_D AND β.
2. CONSTRUCT CIRCLE OF CONVENIENT SIZE TANGENT TO LINE AT ANGLE ϕ_D. (IN THE EXAMPLE SHOWN THE SCALE IS SUCH THAT $\overline{OA} = \sigma_i = \gamma h \cos\beta$, BUT THIS IS NOT ESSENTIAL TO DETERMINATION OF K_A*.)
3. CONSTRUCT VERTICAL LINE FROM THE POLE P_A TO INTERSECT THE CIRCLE AT B.
4. THE DISTANCE $\overline{OB}$ IN THE EXAMPLE EQUALS $\sigma_{\beta MIN}$ AND $\overline{OC} = (\sigma_{\beta MIN}) \cos\beta = \sigma_h$

   THEN $K_A = \dfrac{\sigma_h}{\sigma_i} = \dfrac{\overline{OC}}{\overline{OA}}$
5. THE ANGLE θ_A, WHICH DEFINES THE DIRECTION OF THE ACTIVE FAILURE PLANE, IS THE ANGLE FORMED WITH THE HORIZONTAL BY A LINE FROM THE POLE THROUGH THE POINT OF TANGENCY BETWEEN THE ϕ_D LINE AND THE CIRCLE (POINT D).
6. THE ACTIVE FORCE E_A ACTING ON THE VERTICAL BOUNDARY HAVING DEPTH h BELOW THE SLOPE IS $E_A = \dfrac{\gamma h^2}{2} K_A$

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* IT IS APPARENT THAT THE RATIO $\dfrac{\overline{OC}}{\overline{OA}}$ IS INDEPENDENT OF THE SCALE USED IN CONSTRUCTING THE CIRCLE.

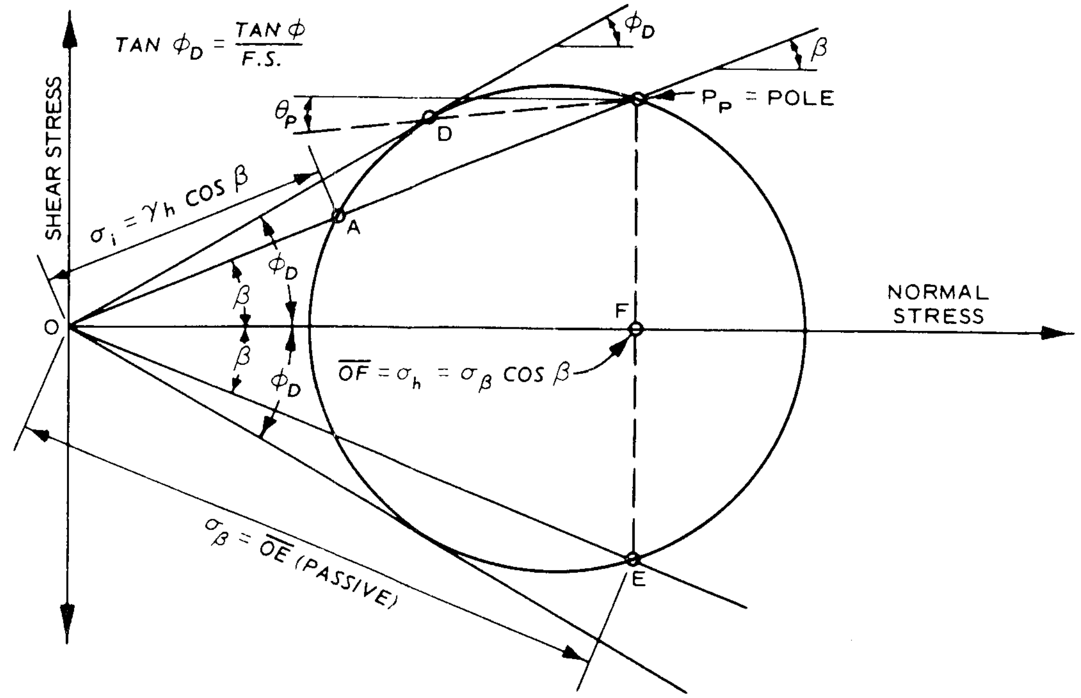


FIGURE 3. PROCEDURE FOR DETERMINING K_P & θ_P, PASSIVE CASE (σ_β = MAXIMUM)

1. CONSTRUCT LINES AT ANGLES ϕ_D AND β.
2. CONSTRUCT CIRCLE OF CONVENIENT SIZE TANGENT TO LINE AT ANGLE ϕ_D. (IN THE EXAMPLE SHOWN, THE SCALE CHOSEN IS SUCH THAT $\overline{OA} = \sigma_i = \gamma h \cos\beta$, BUT THIS IS NOT ESSENTIAL TO DETERMINATION OF K_P.*)
3. CONSTRUCT VERTICAL LINE FROM POLE P_P TO INTERSECT CIRCLE AT E.
4. THE DISTANCE $\overline{OE}$ IN THE EXAMPLE EQUALS $\sigma_{\beta MAX}$ AND $\overline{OF} = (\sigma_{\beta MAX}) \cos\beta = \sigma_h$.

   THEN $K_P = \dfrac{\sigma_h}{\sigma_i} = \dfrac{\overline{OF}}{\overline{OA}}$
5. THE ANGLE θ_P, WHICH DEFINES THE DISTANCE OF THE PASSIVE FAILURE PLANE IS THAT ANGLE FORMED WITH THE HORIZONTAL BY A LINE FROM THE POLE P_P THROUGH THE POINT OF TANGENCY BETWEEN THE ϕ_D LINE AND THE CIRCLE (POINT D).
6. THE PASSIVE FORCE E_P ACTING ON THE VERTICAL BOUNDARY HAVING DEPTH h BELOW THE SLOPE IS $E_P = \dfrac{\gamma h^2}{2} K_P$.

---

* IT IS APPARENT THAT THE RATIO $\dfrac{\overline{OF}}{\overline{OA}}$ IS INDEPENDENT OF THE SCALE USED IN CONSTRUCTING THE CIRCLE.

USE OF CONJUGATE STRESSES

ADOPTED DESIGN DATA							
MATERIAL	UNIT WT LB/CU FT		ϕ DEG		c KIPS/SQ FT		
	γ_m	γ'	R*	S	R	S	
COHESIONLESS EMBANKMENT SHELL AND FOUNDATION	130	77	—	35	—	0	
SOFT CLAY FOUNDATION*	—	—	14	—	0.3	—	

* ASSUMED FULLY CONSOLIDATED UNDER EMBANKMENT, THUS Q STRENGTH NOT APPLICABLE.

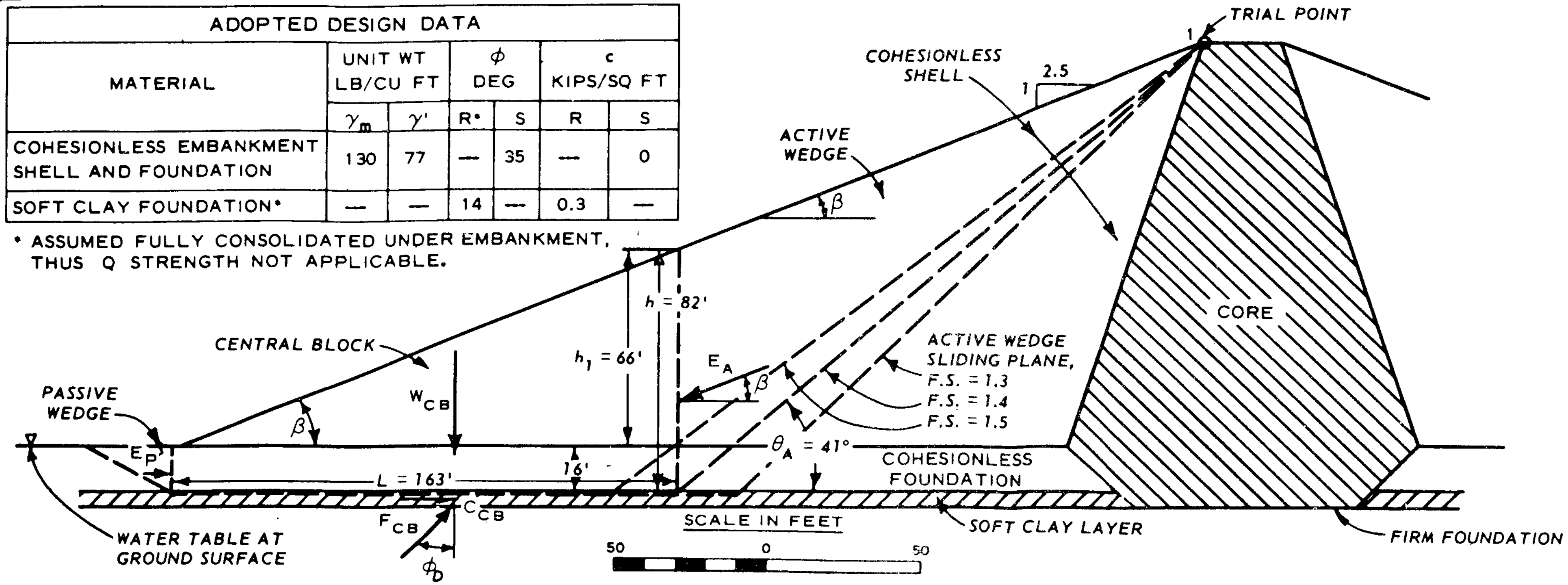
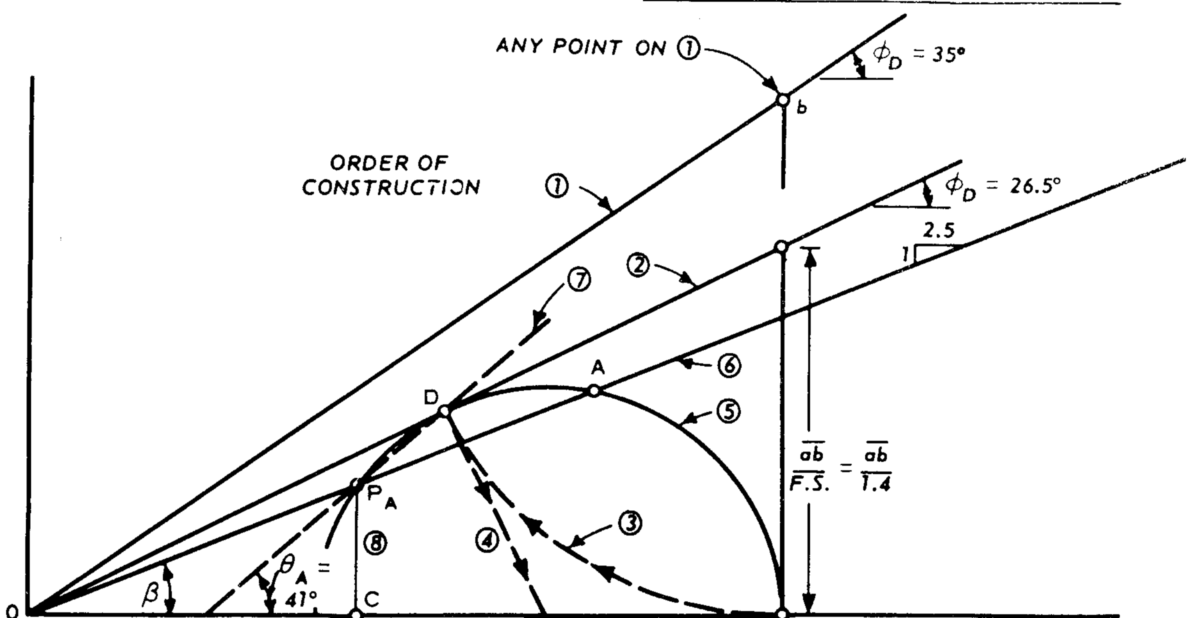


FIGURE 1. EMBANKMENT SECTION



$$K_A = \frac{\overline{OC}}{\overline{OA}} = 0.538$$

$$E_A = \left(\frac{\gamma_m h^2}{2}\right)^* K_A$$

$$E_A = \left(\frac{0.13(82)^2}{2}\right)0.538$$

E_A = 235 KIPS

$$K_P = \frac{1 + \sin \phi_D}{1 - \sin \phi_D} = \frac{1.446}{0.554} = 2.61$$

$$E_P = \frac{\gamma' h_2^2}{2} K_P = \frac{0.077(16)^2}{2} \times 2.61$$

E_P = 26 KIPS

FIGURE 2. DETERMINATION OF E_A & E_P, TRIAL F.S. = 1.4

* THE EFFECT OF SUBMERGENCE OF THE LOWER PORTION OF THE ACTIVE WEDGE IN THIS CASE IS INSIGNIFICANT AND IGNORED HERE. IF AN APPRECIABLE PORTION OF THE ACTIVE WEDGE IS SUBMERGED AS IN CASE IV, PARTIAL POOL, IT IS EASIER TO DETERMINE E_A BY CONSTRUCTING A FORCE POLYGON.

$$W_{CB} = \frac{h_1 \times L}{2}\gamma_m + h_2 L \gamma'$$

$$= \left(\frac{66 \times 163}{2} \times 0.13\right) + (16 \times 163 \times 0.077)$$

= 901 KIPS

$$C_{CB} = C_D \times L = \frac{0.3}{1.4} \times 163 = 35 \text{ KIPS}$$

$$\phi_D \text{ (CLAY)} = \text{ARC TAN } \frac{0.249}{1.4} = 10.1°$$

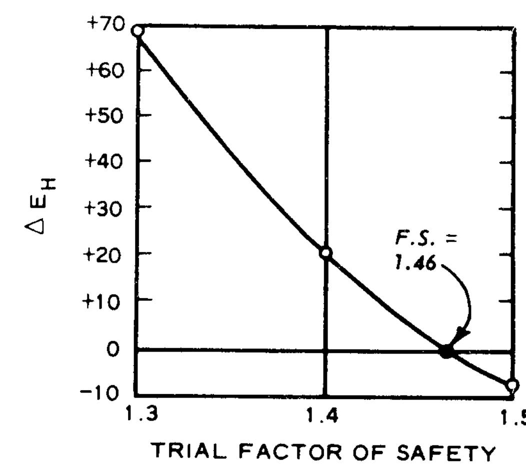


FIGURE 4. TRIAL F.S. VS ΔE

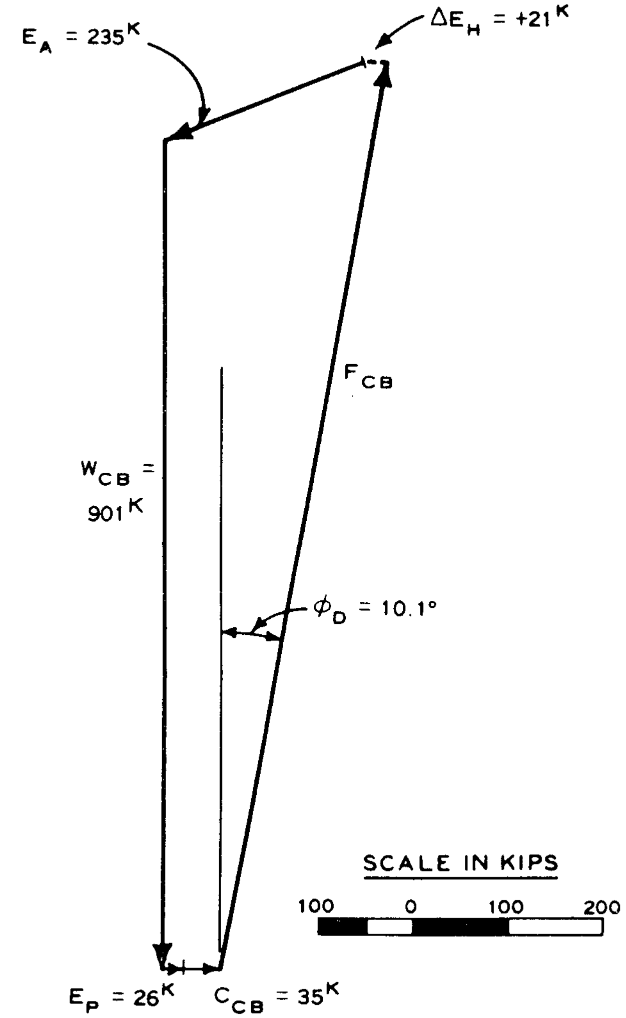


FIGURE 3. FORCE POLYGON, CENTRAL BLOCK TRIAL F.S. = 1.4

STABILITY ANALYSIS OF EMBANKMENT WITH CENTRAL CORE, CASE I - END OF CONSTRUCTION, WEDGE METHOD

ADOPTED DESIGN DATA					
MATERIAL	UNIT WT LB/CU FT	ϕ DEG		c. KIPS/SQ FT	
	γ_m	Q	S	Q	S.
ROCKFILL	116	—	40	—	0
GRAVEL FILTER	133*	—	35*	—	0
CORE	146	17	—	0.7	—

* ASSUMED SAME AS ROCKFILL IN DETERMINING E_P.

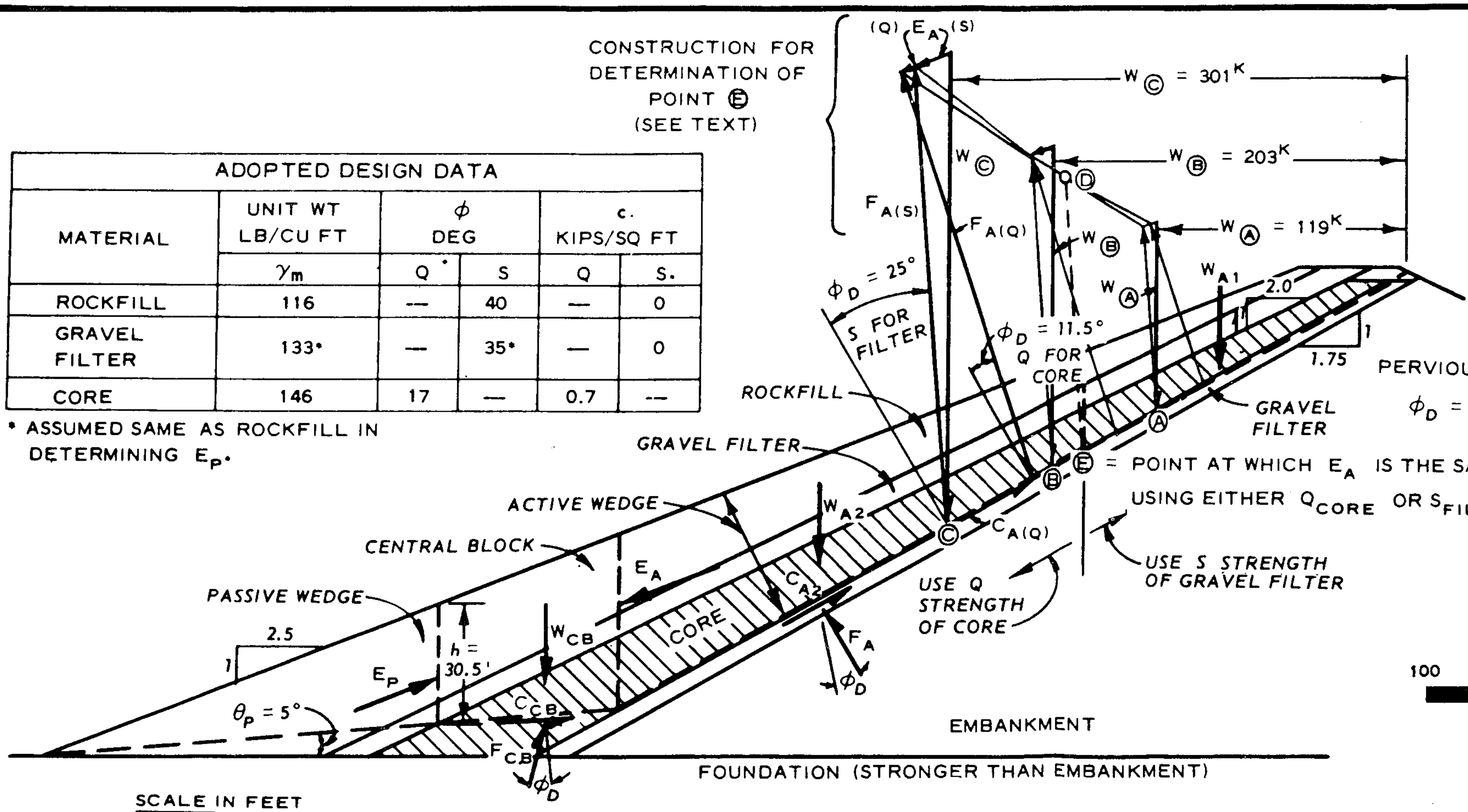


FIGURE 1. EMBANKMENT SECTION

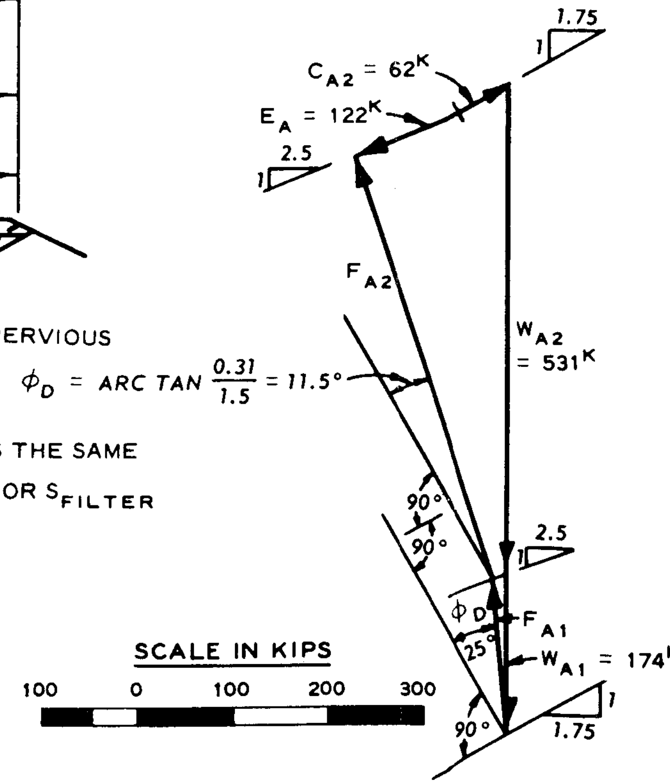


FIGURE 3. FORCE POLYGON ACTIVE WEDGE, TRIAL F.S. = 1.5

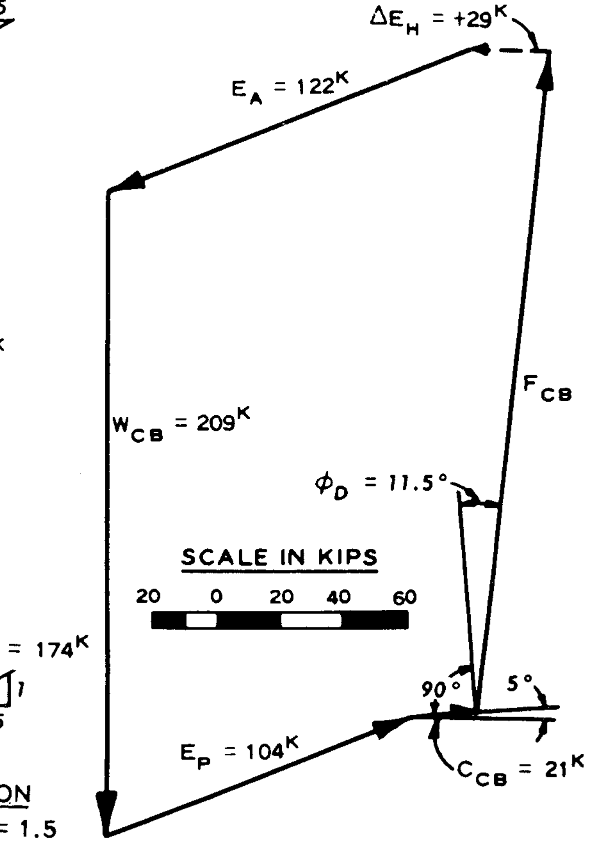
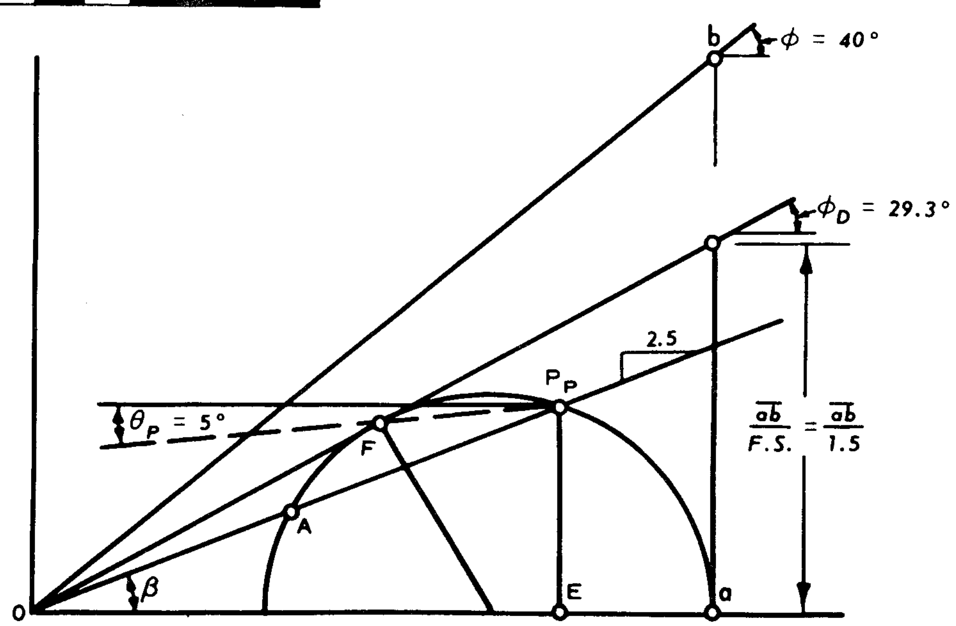


FIGURE 4. FORCE POLYGON, CENTRAL BLOCK, TRIAL F.S. = 1.5



$K_P = \frac{\overline{OE}}{\overline{OA}} = 1.92$ $\qquad E_P = \frac{1}{2}\gamma_m h^2 K_P = \frac{1}{2}(0.116)(30.5)^2(1.92) = 104$ KIPS

FIGURE 2. DETERMINATION OF E_P & θ_P, TRIAL F.S. = 1.5

DETERMINATION OF W_{A1} & W_{A2}

	VOL A1 FT^3	VOL A2 FT^3	γ_m KCF	W_{A1} KIPS	W_{A2} KIPS
ROCKFILL	334	1682	0.116	38.8	195.2
FILTER	346	633	0.133	46.0	84.2
CORE	610	1720	0.146	89.0	252.0
		TOTAL:		173.8K	531.4K

DETERMINATION OF W_{CB}

ROCKFILL	1002 FT^3 × 0.116 KCF = 116.3K
FILTER	245 FT^3 × 0.133 KCF = 32.6
CORE	412 FT^3 × 0.146 KCF = 60.1
TOTAL	= 209.0K

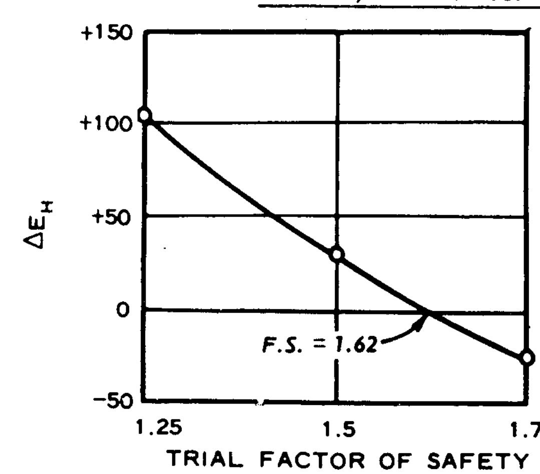


FIGURE 5. TRIAL F.S. VS ΔE

STABILITY ANALYSIS OF EMBANKMENT WITH INCLINED CORE, CASE I - END OF CONSTRUCTION, WEDGE METHOD

ADOPTED DESIGN DATA						
MATERIAL	UNIT WT LB/CU FT		TAN ϕ		c KIPS/SQ FT	
	γ_{SAT}	γ'	R	S	R	S
SHELL AND COHESIONLESS FOUNDATION	140	78	0.466	0.700	1.0	0
CLAY FOUNDATION	---	---	0.344	0.577	0.7	0

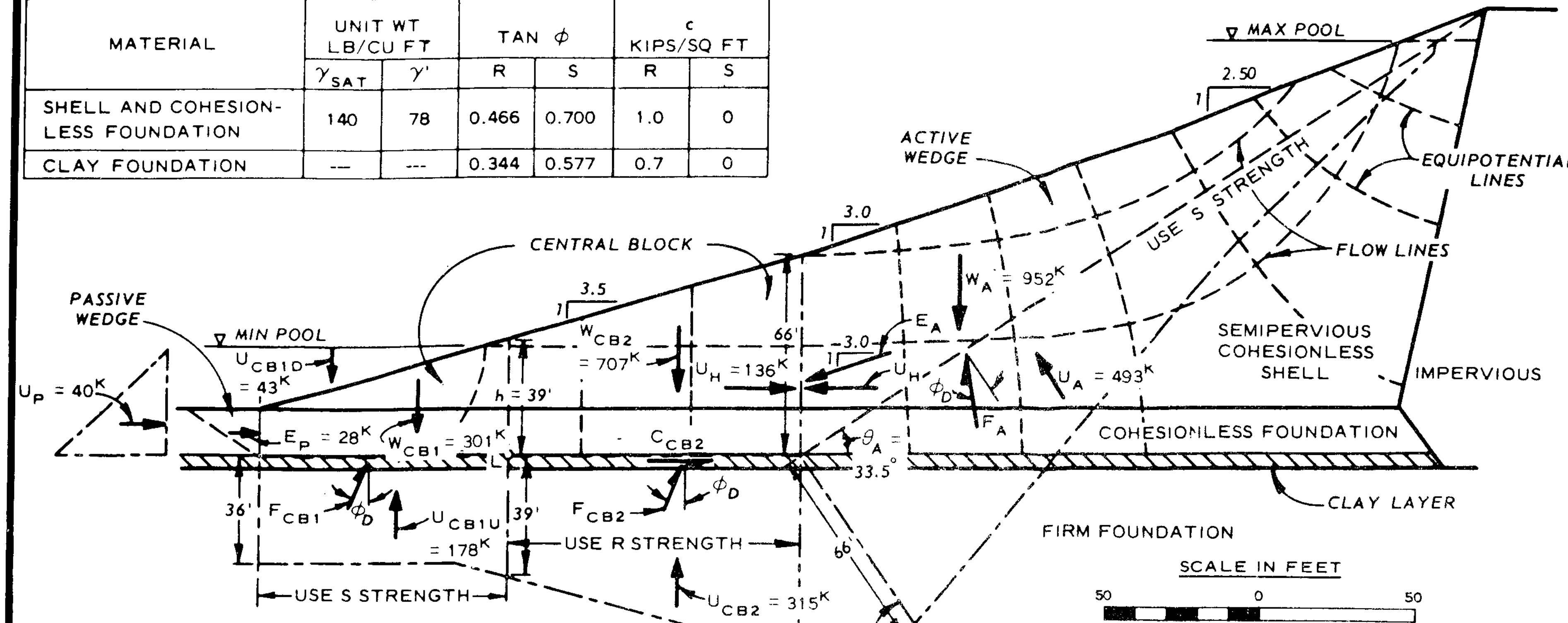


FIGURE 1. EMBANKMENT SECTION (WITH FLOW NET) (TRIAL F.S. = 1.3)

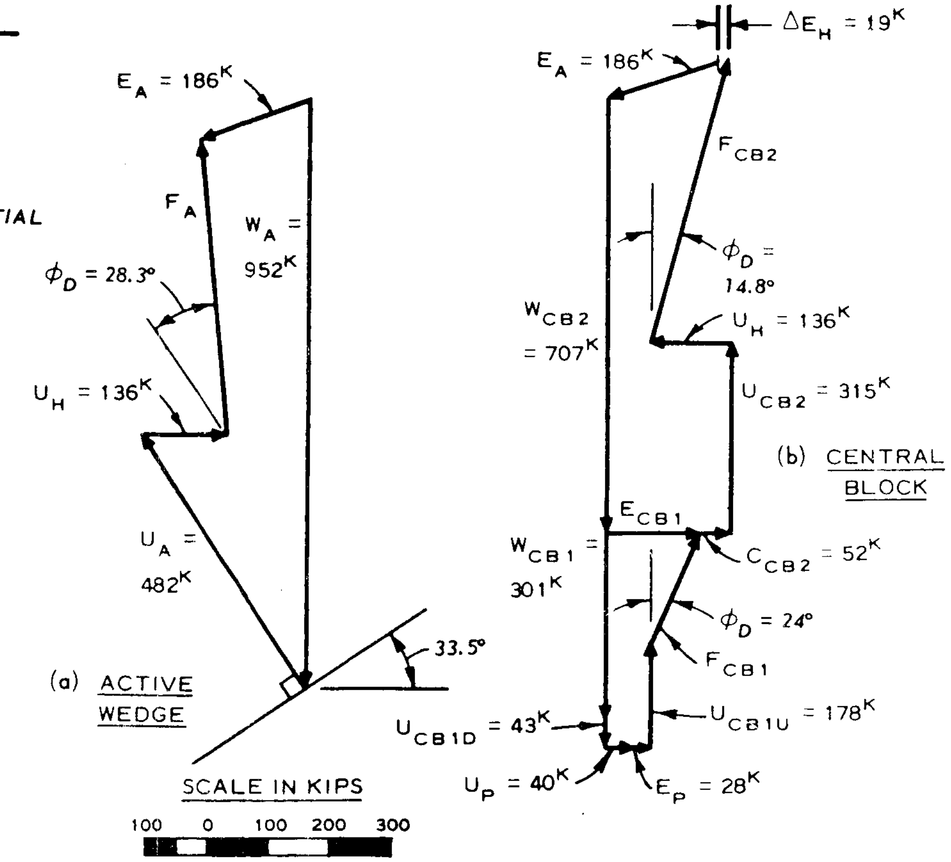


FIGURE 3. ACTIVE WEDGE AND CENTRAL BLOCK FORCE POLYGONS, TRIAL F.S. = 1.3

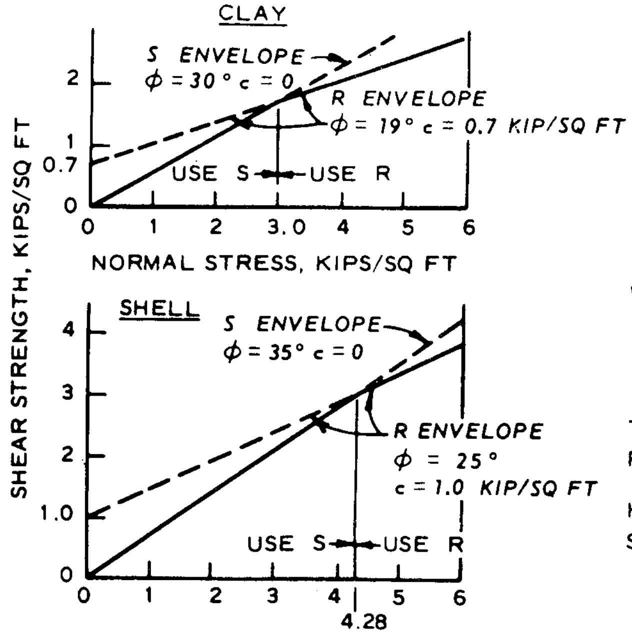


FIGURE 2. COMPOSITE STRENGTH ENVELOPES

$$E_P = \frac{1}{2}\gamma' h^2 K_P = \frac{(0.078)(16)^2}{2}\left[\frac{1 + \sin\left(\text{ARC TAN } \frac{0.70}{1.3}\right)}{1 - \sin\left(\text{ARC TAN } \frac{0.70}{1.3}\right)}\right] = 28 \text{ KIPS}$$

SHEAR STRENGTHS ALONG CENTRAL BLOCK AND ACTIVE WEDGE TRIAL FAILURE PLANES:

HEIGHT h AT LOCATION ALONG CENTRAL BLOCK SLIDING PLANE WHERE EFFECTIVE STRESS IS 3.0 KIPS/SQ FT IS

$$h = \frac{3.0}{\gamma'} = \frac{3.0 \text{ KIPS/SQ FT}}{0.078 \text{ KIP/CU FT}} = 39 \text{ FT}$$

THEREFORE USE S STRENGTH TO LEFT OF L (WHERE h = 39 FT) AND R STRENGTH TO RIGHT OF L.

HEIGHT h ALONG ACTIVE WEDGE SLIDING PLANE WHERE EFFECTIVE STRESS IS 4.28 KIPS/SQ FT IS

$$h = \frac{4.28 \text{ KIPS/SQ FT}}{\gamma' \times \cos\theta_A} = \frac{4.28 \text{ KIPS/SQ FT}}{0.078 \text{ KIP/CU FT} \times \cos 33.5°}$$

= 65.8 FT SINCE MAXIMUM HEIGHT IS 66 FT, USE S STRENGTH ALONG ENTIRE PLANE.

NOTE: THE COMPUTATIONS ABOVE ARE BASED ON VERTICAL EQUIPOTENTIAL LINES FOR SIMPLICITY.

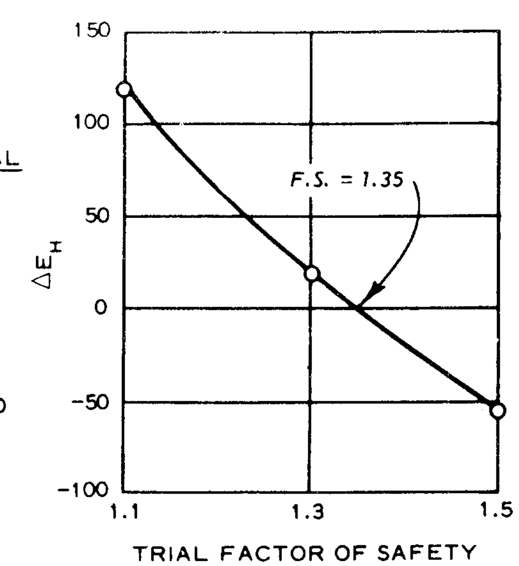


FIGURE 4. TRIAL F.S. VS ΔE_H

NOTE: W_A, W_{CB1}, AND W_{CB2} ARE BASED ON SATURATED WEIGHTS.

STABILITY ANALYSIS, EMBANKMENT WITH CENTRAL CORE AND SEMIPERVIOUS SHELL, CASE II - SUDDEN DRAWDOWN. WEDGE METHOD

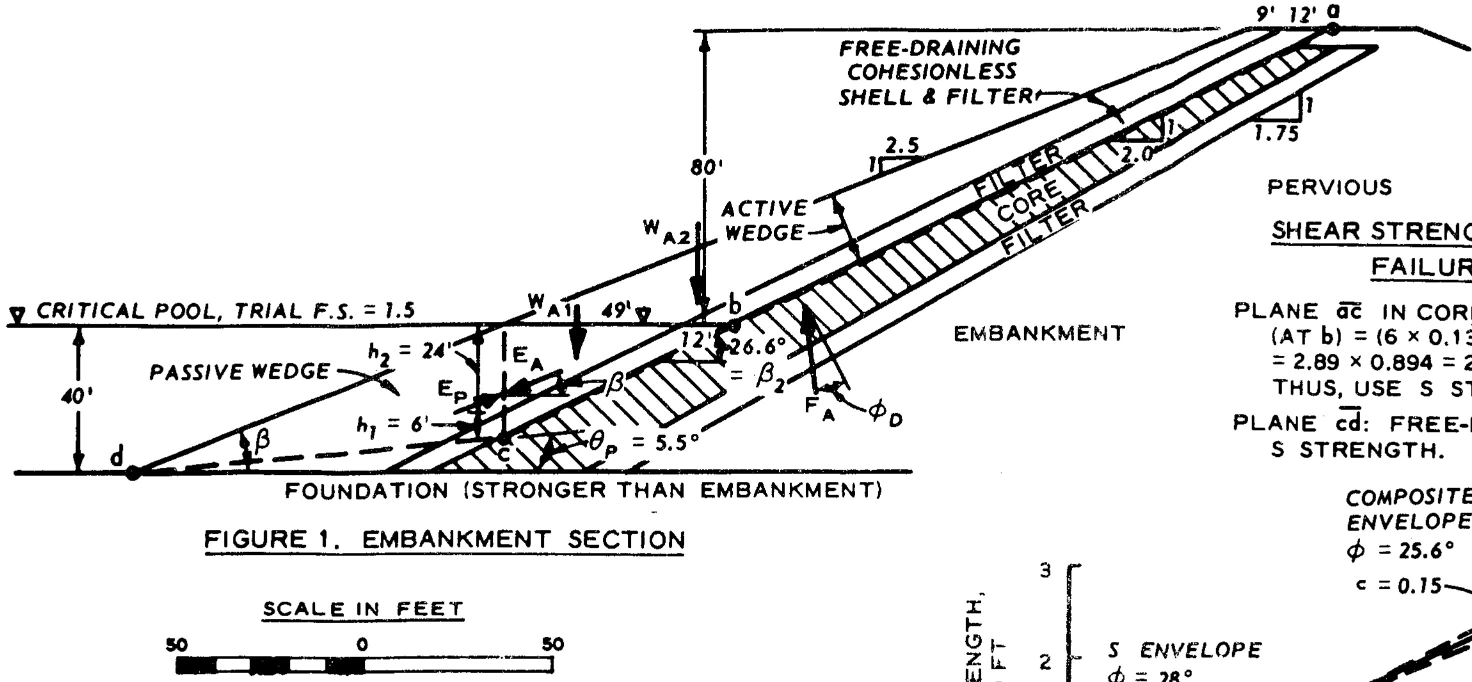


FIGURE 1. EMBANKMENT SECTION

SHEAR STRENGTH ALONG TRIAL FAILURE PLANES:

PLANE $\overline{ac}$ IN CORE: MAX NORMAL STRESS (AT b) = (6 × 0.133 + 18 × 0.116) COS 26.6° = 2.89 × 0.894 = 2.59 KIPS/SQ FT < 2.83 THUS, USE S STRENGTH.

PLANE $\overline{cd}$: FREE-DRAINING SHELL, USE S STRENGTH.

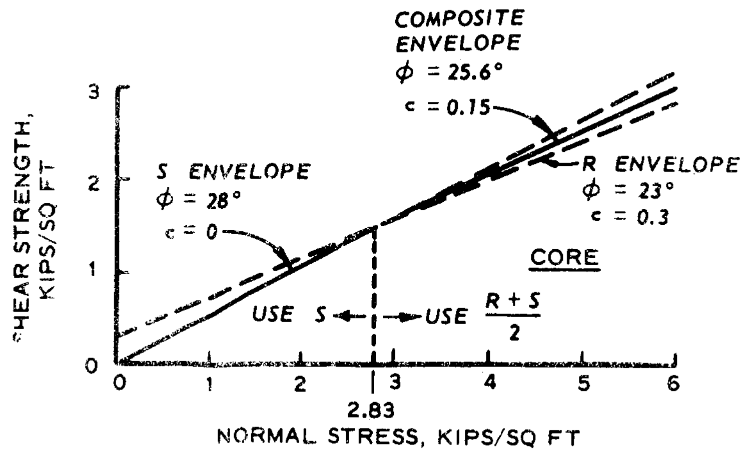
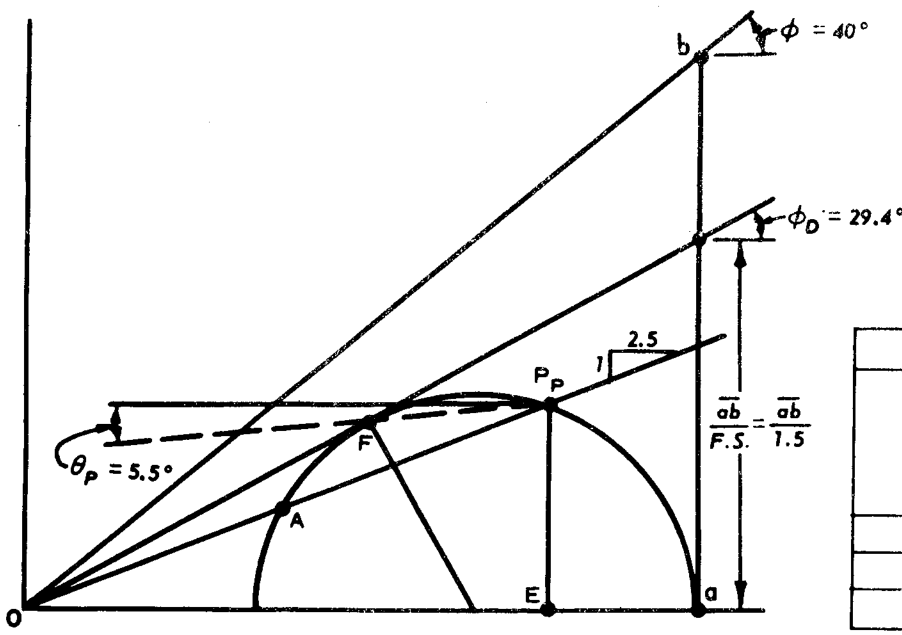


FIGURE 3. COMPOSITE STRENGTH ENVELOPE, CORE



$$K_P = \frac{\overline{OE}}{\overline{OA}} = 1.90$$

$$E_P = \left[\frac{\gamma_2'(h_2)^2}{2} + h_1\gamma_2' h_2 + \frac{\gamma_1'(h_1)^2}{2}\right] K_P$$

$$E_P = \left[\frac{0.073}{2} \times (24)^2 + 6(0.073)(24) + \frac{0.079(6)^2}{2}\right] \times 1.90 = 62^K$$

FIGURE 2. DETERMINATION OF E_P & θ_P, TRIAL F.S. = 1.5

ADOPTED DESIGN VALUES								
MATERIAL	UNIT WT LB/CU FT		TAN ϕ			c KIPS/SQ FT		
	γ_m	γ'	R	S	$\frac{R+S}{2}$	R	S	$\frac{R+S}{2}$
SHELL	116	73	---	0.839	---	---	0	---
CORE	146	87	0.424	0.532	0.478	0.3	0	0.15
FILTER	133	79	SAME AS SHELL					

DETERMINATION OF WEIGHTS, ACTIVE WEDGE

$$W_{A1} = \frac{1}{2} \times 24 \times 49 \times 0.073 + \left(\frac{1}{2} \times 30 \times 61 - \frac{1}{2} \times 24 \times 49\right) 0.079 = 69^K$$

$$W_{A2} = \frac{49+9}{2} \times 80 \times 0.116 + 12 \times 80 \times 0.133 = 397^K$$

$$W_A = 466^K$$

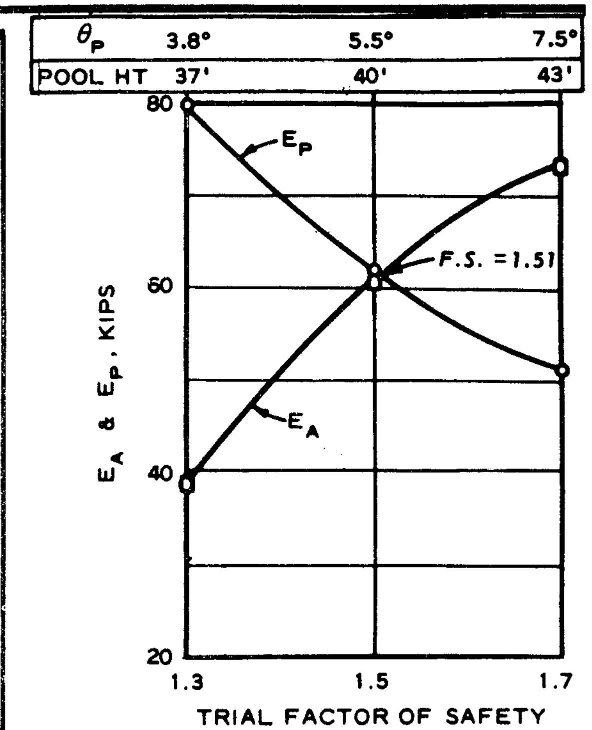


FIGURE 5. TRIAL FACTOR OF SAFETY VS E_A & E_P

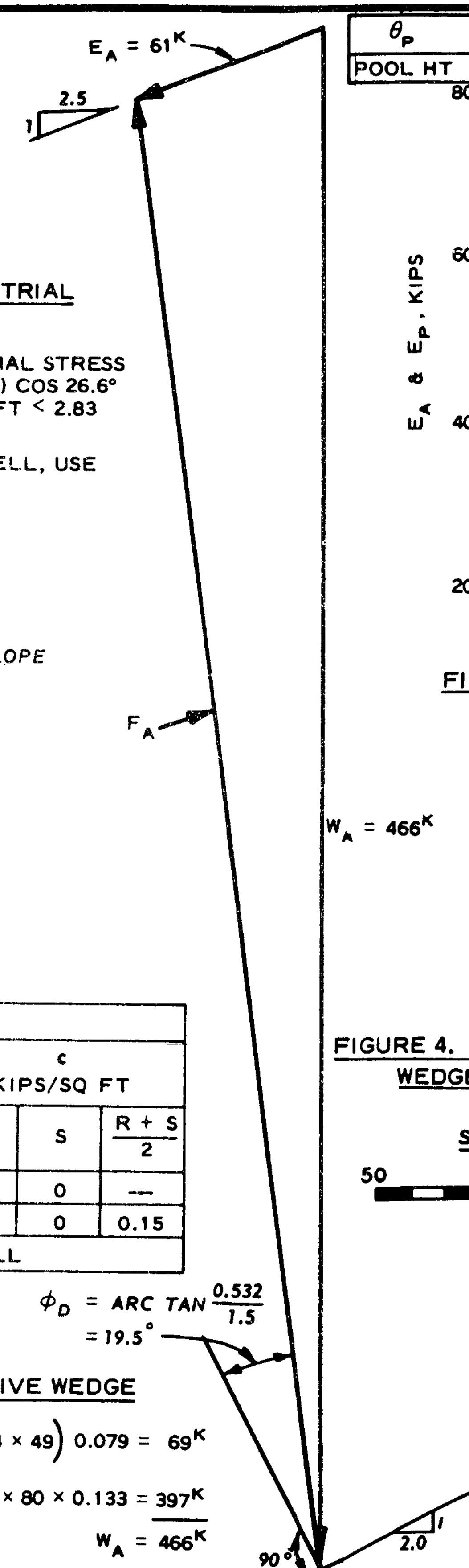
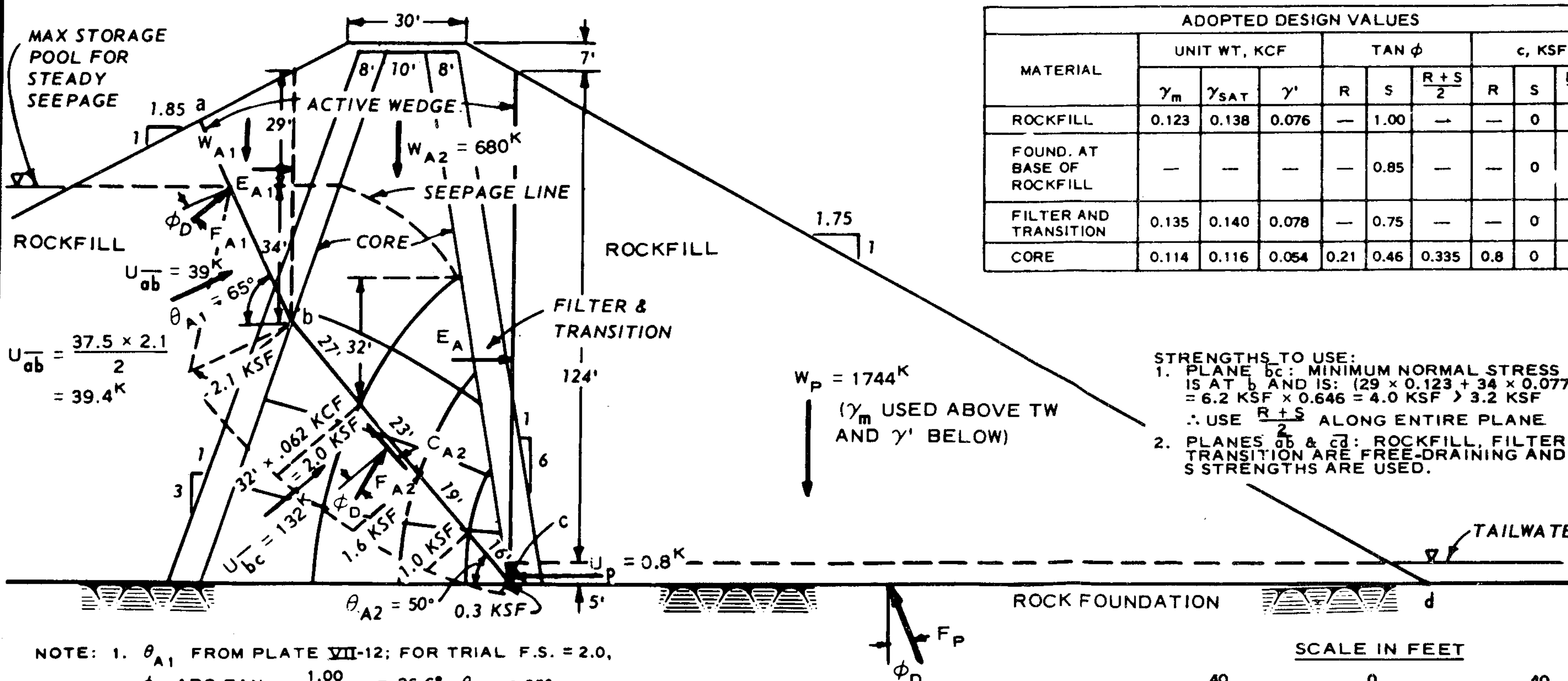


FIGURE 4. FORCE POLYGON, ACTIVE WEDGE FOR TRIAL F.S. = 1.5

STABILITY ANALYSIS, EMBANKMENT WITH INCLINED CORE AND FREE-DRAINING SHELL, CASE IV-PARTIAL POOL, WEDGE METHOD



MATERIAL	UNIT WT, KCF			TAN ϕ			c, KSF		
	γ_m	γ_{SAT}	γ'	R	S	$\frac{R+S}{2}$	R	S	$\frac{R+S}{2}$
ROCKFILL	0.123	0.138	0.076	—	1.00	—	—	0	—
FOUND. AT BASE OF ROCKFILL	—	—	—	—	0.85	—	—	0	—
FILTER AND TRANSITION	0.135	0.140	0.078	—	0.75	—	—	0	—
CORE	0.114	0.116	0.054	0.21	0.46	0.335	0.8	0	0.4

ADOPTED DESIGN VALUES

STRENGTHS TO USE:
1. PLANE $\overline{bc}$: MINIMUM NORMAL STRESS IS AT b AND IS: (29 × 0.123 + 34 × 0.077) COS 50° = 6.2 KSF × 0.646 = 4.0 KSF > 3.2 KSF
∴ USE $\frac{R+S}{2}$ ALONG ENTIRE PLANE
2. PLANES $\overline{ab}$ & $\overline{cd}$: ROCKFILL, FILTER AND TRANSITION ARE FREE-DRAINING AND S STRENGTHS ARE USED.

NOTE: 1. θ_{A1} FROM PLATE VII-12; FOR TRIAL F.S. = 2.0,
ϕ_D ARC TAN $\frac{1.00}{F.S. = 2.0}$ = 26.6°, θ_{A1} = 65°
2. θ_{A2} ASSUMED = 50° FOR FIRST TRIAL
3. W_{A1} AND W_{A2} CALCULATED USING γ_m ABOVE SEEPAGE LINE AND γ_{SAT} BELOW

FIGURE 1. EMBANKMENT SECTION

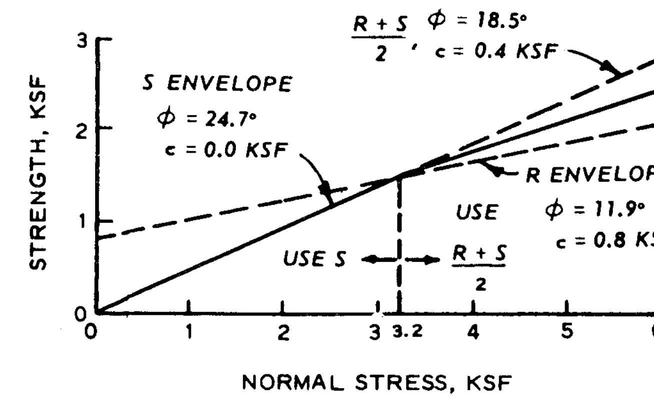


FIGURE 2. COMPOSITE STRENGTH ENVELOPE, CORE

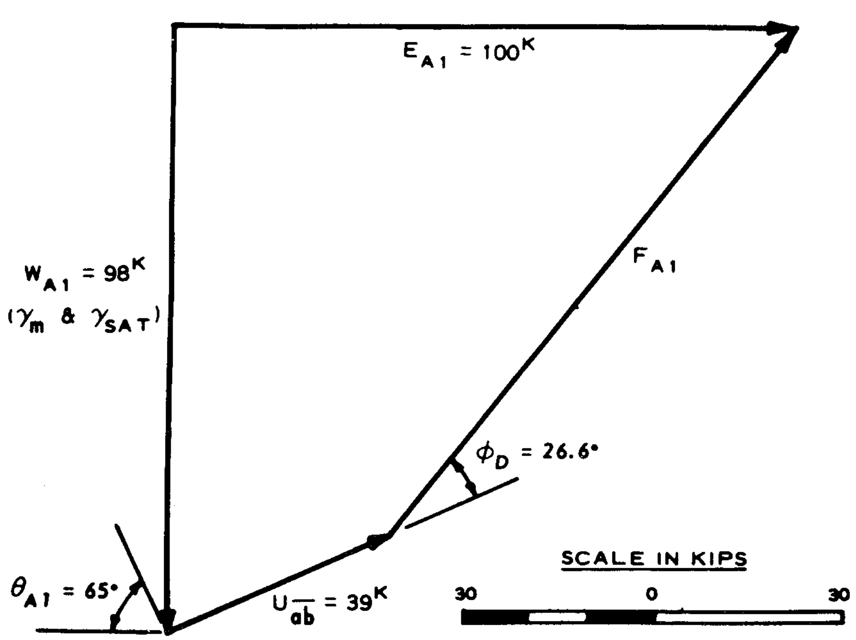


FIGURE 3. DETERMINATION OF E_{A1}, θ_{A2} = 50°, F.S. = 2.0

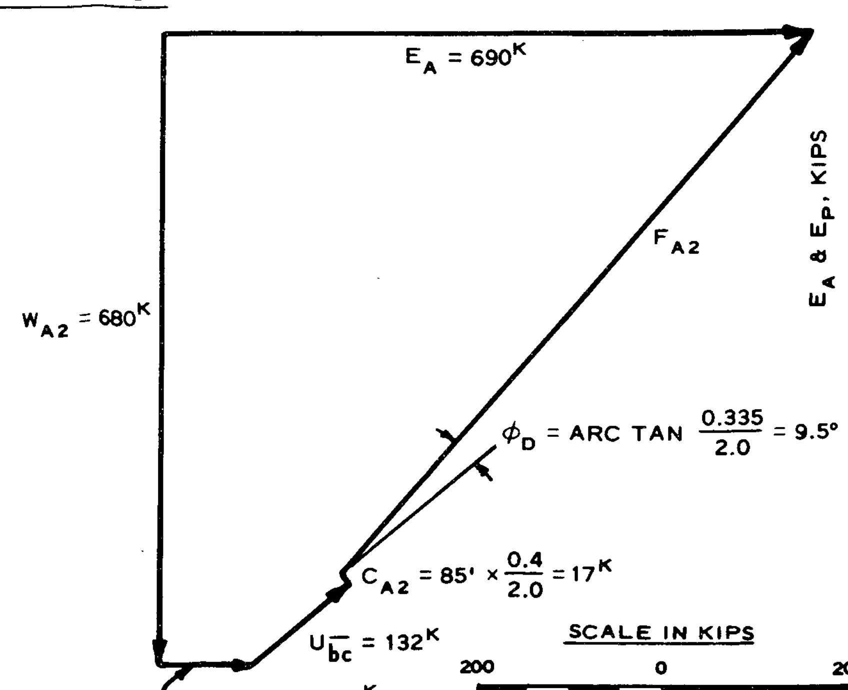


FIGURE 4. DETERMINATION OF E_A, TRIAL F.S. = 2.0

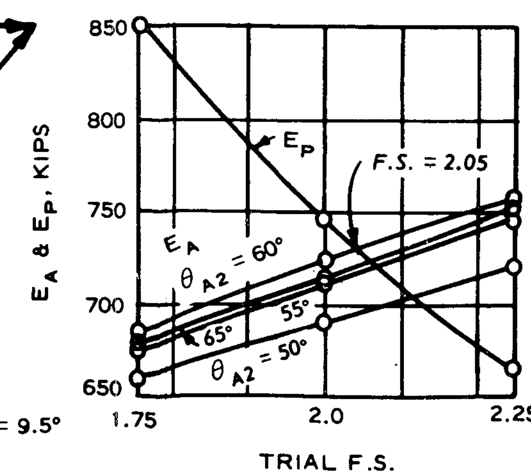


FIGURE 6. E_A & E_P VERSUS TRIAL F.S. AND θ_{A2}

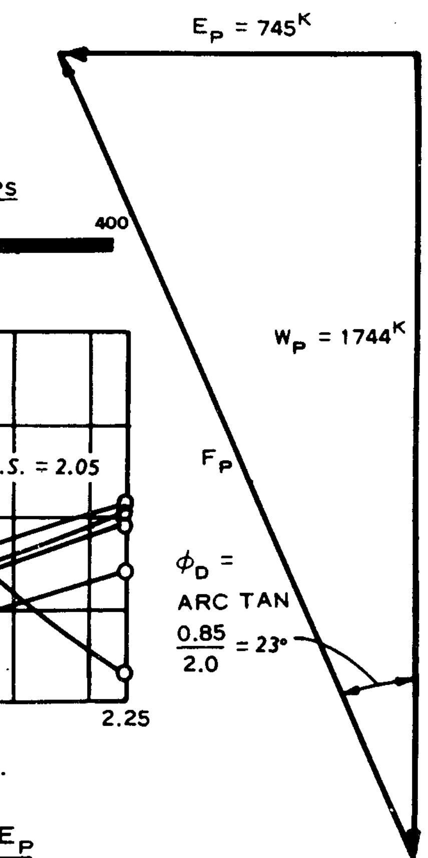
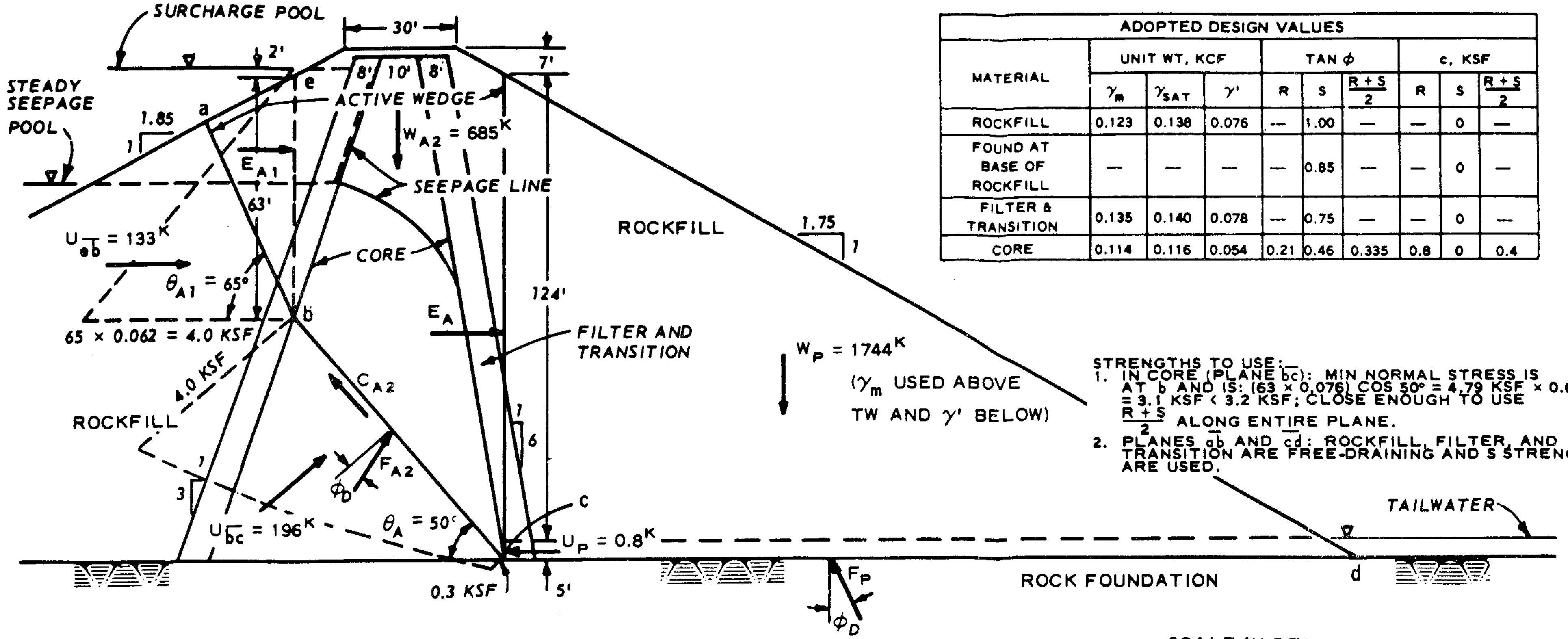


FIGURE 5. DETERMINATION OF E_P, F.S. = 2.0

STABILITY ANALYSIS, EMBANKMENT WITH CENTRAL CORE, CASE V-STEADY SEEPAGE, WEDGE METHOD



ADOPTED DESIGN VALUES									
MATERIAL	UNIT WT, KCF			TAN ϕ			c, KSF		
	γ_m	γ_{SAT}	γ'	R	S	$\frac{R+S}{2}$	R	S	$\frac{R+S}{2}$
ROCKFILL	0.123	0.138	0.076	—	1.00	—	—	0	—
FOUND AT BASE OF ROCKFILL	—	—	—	—	0.85	—	—	0	—
FILTER & TRANSITION	0.135	0.140	0.078	—	0.75	—	—	0	—
CORE	0.114	0.116	0.054	0.21	0.46	0.335	0.8	0	0.4

STRENGTHS TO USE:

1. IN CORE (PLANE $\overline{bc}$): MIN NORMAL STRESS IS AT b AND IS: (63 × 0.076) COS 50° = 4.79 KSF × 0.646 = 3.1 KSF < 3.2 KSF; CLOSE ENOUGH TO USE $\frac{R+S}{2}$ ALONG ENTIRE PLANE.
2. PLANES $\overline{ab}$ AND $\overline{cd}$: ROCKFILL, FILTER, AND TRANSITION ARE FREE-DRAINING AND S STRENGTHS ARE USED.

SCALE IN FEET
40 0 40

FIGURE 1. EMBANKMENT SECTION

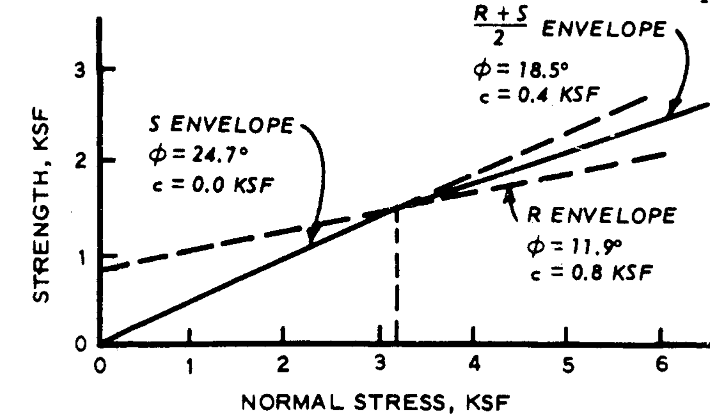


FIGURE 2. COMPOSITE STRENGTH ENVELOPE, CORE

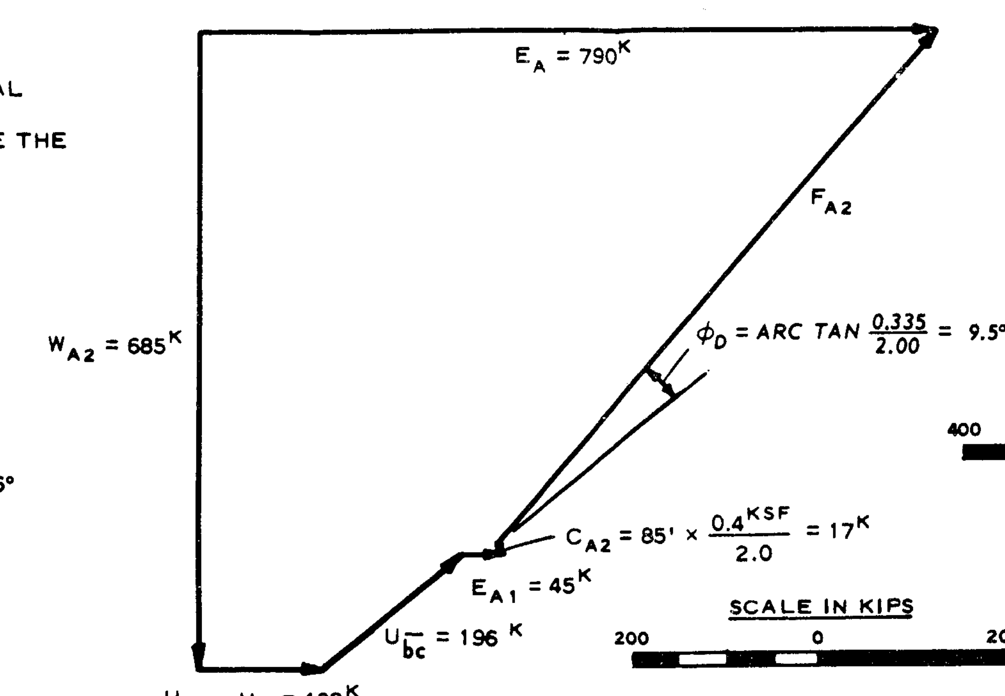
NOTE:
1. θ_{A1} FROM PLATE VII-11; FOR TRIAL F.S. = 2.0, ϕ_D ARC TAN $\frac{1.00}{2.0}$ = 26.6°, θ_{A1} = 65°
2. θ_{A2} ASSUMED AS 50° FOR FIRST TRIAL
3. W_{A2} CALCULATED USING γ_m ABOVE THE SEEPAGE LINE AND γ_{SAT} BELOW

$$E_{A1} = \frac{1}{2}\gamma' h^2 K_A$$

FOR TRIAL F.S. = 2.0, ϕ_D = ARC TAN $\frac{1.00}{2}$ = 26.6°

K_A FROM PLATE VII-12 = 0.297

$$E_{A1} = \frac{1}{2} \times 0.076 \times 63^2 \times 0.297 = 45^K$$



SCALE IN KIPS
200 0 200

FIGURE 3. DETERMINATION OF E_A, TRIAL F.S. = 2.0

SCALE IN KIPS
400 0 400

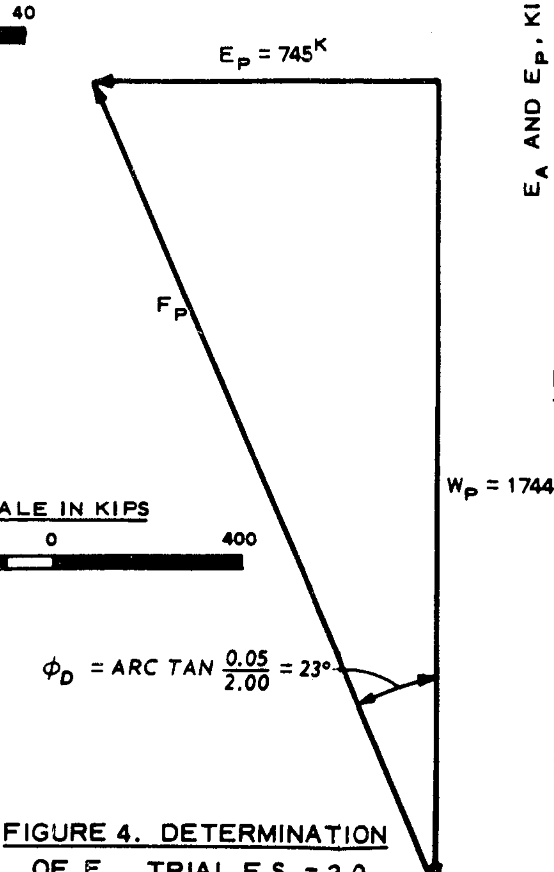


FIGURE 4. DETERMINATION OF E_P, TRIAL F.S. = 2.0

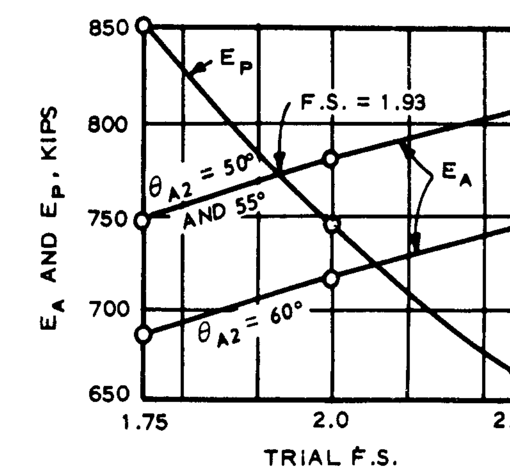
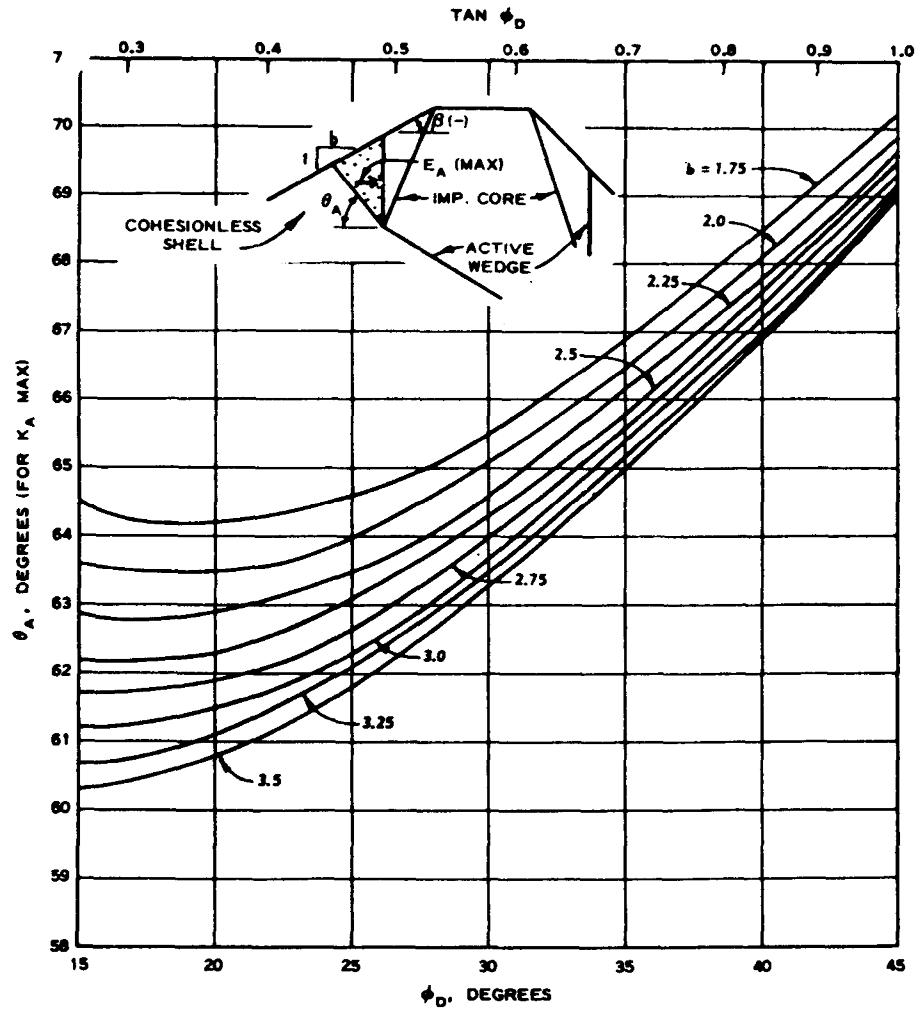


FIGURE 5. E_A AND E_P VERSUS TRIAL F.S. AND θ_{A2}

STABILITY ANALYSIS EMBANKMENT WITH CENTRAL CORE, CASE VI-SURCHARGE POOL, WEDGE METHOD



$$\theta_A = \phi_D + \text{ARC TAN}\left(-\text{TAN}(\phi_D + \beta) + \sqrt{\text{TAN}(\phi_D + \beta)\,[\text{TAN}(\phi_D + \beta) + \text{COT}\,\phi_D]}\right)\dagger$$

† FROM TABLE A-73, JUMIKIS EARTH PRESSURE COEFFICIENT TABLES (1962).[14]

θ_A VS ϕ_D FOR
COHESIONLESS SOIL,
COULOMB ACTIVE SLIDING PLANE
FOR ACTIVE WEDGE BENEATH NEGATIVE SLOPE

Plate VII-11